HERMES

Efficient Ring Packing using MLWE Ciphertexts and Application to Transciphering

Youngjin Bae, Jung Hee Cheon, Jaehyung Kim,
Jai Hyun Park, Damien Stehlé

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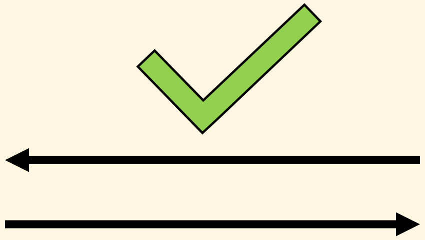

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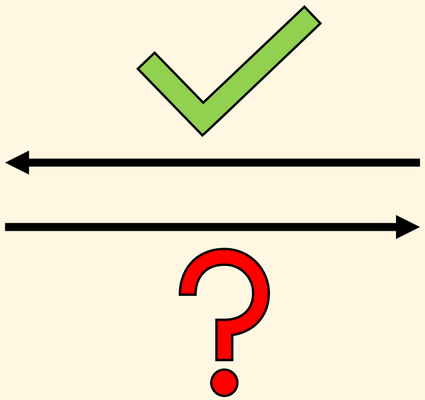

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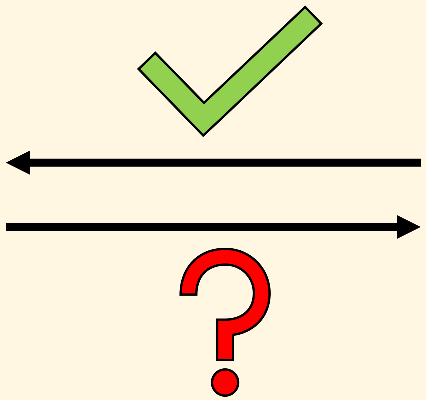

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- Ring packing (RP) bridges LWE and RLWE formats [CGGI17, MS18, BGGJ20, CDKS21, LHH+21]
  - Scheme switching during homomorphic computation
  - Transciphering

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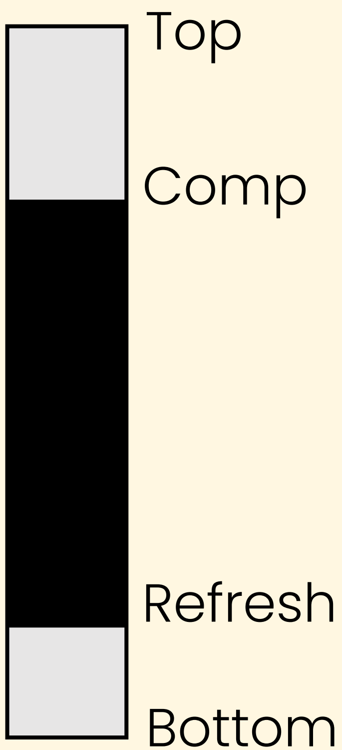

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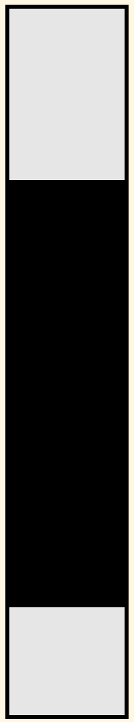

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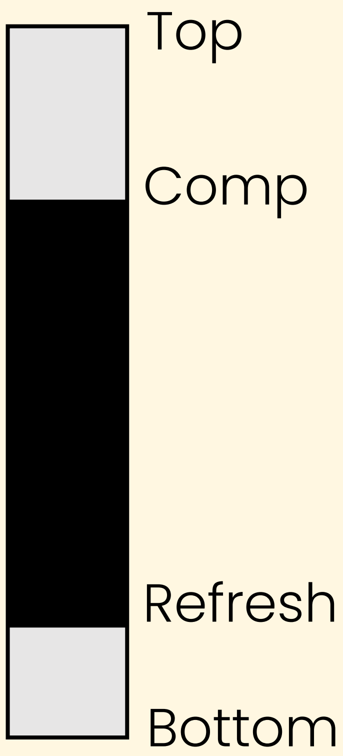

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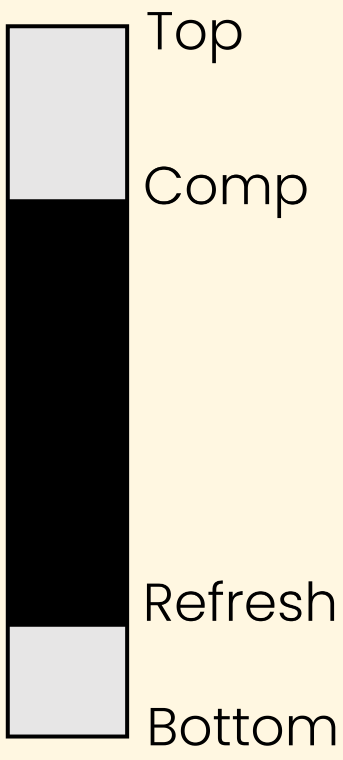

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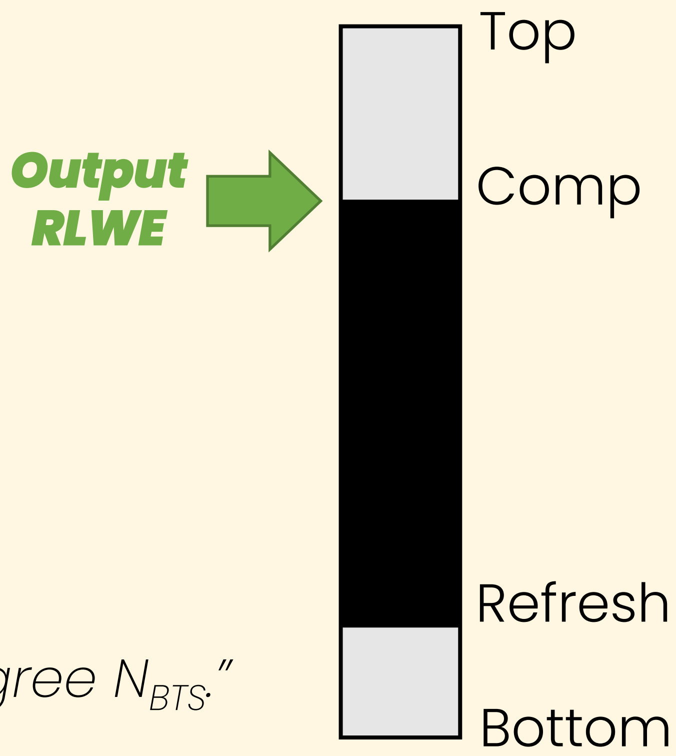
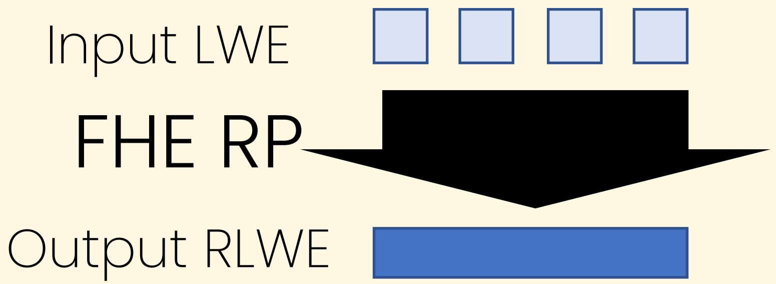
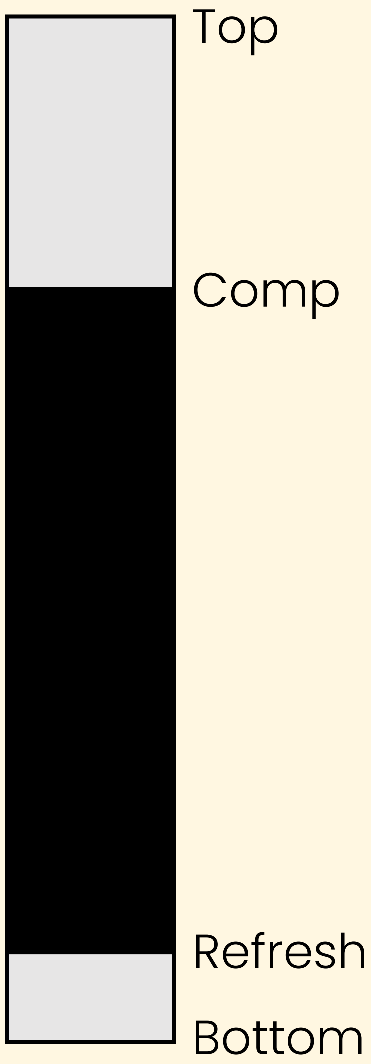


Large parameters for FHE RP





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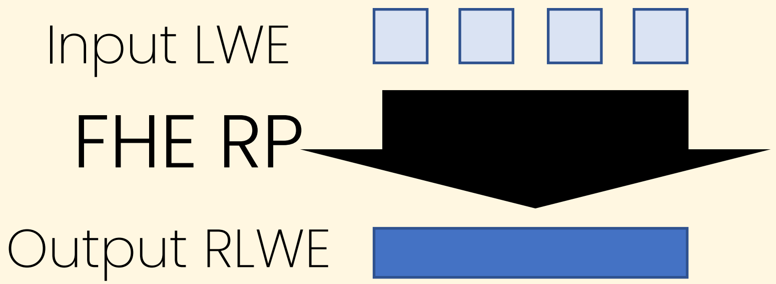

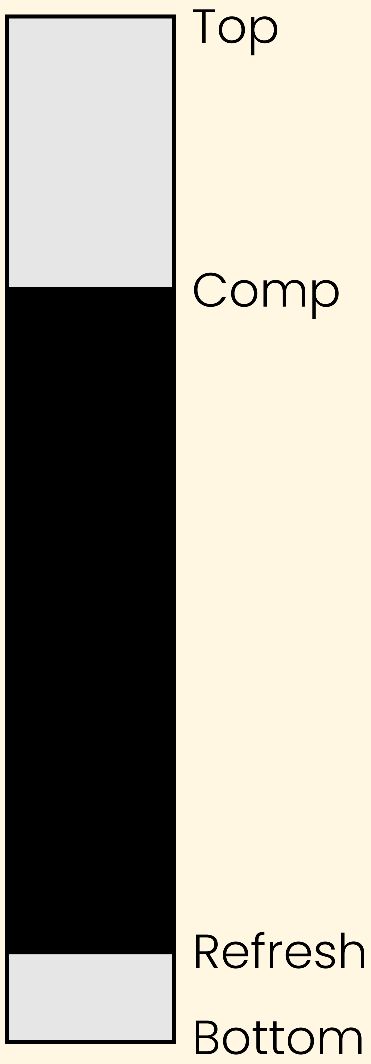

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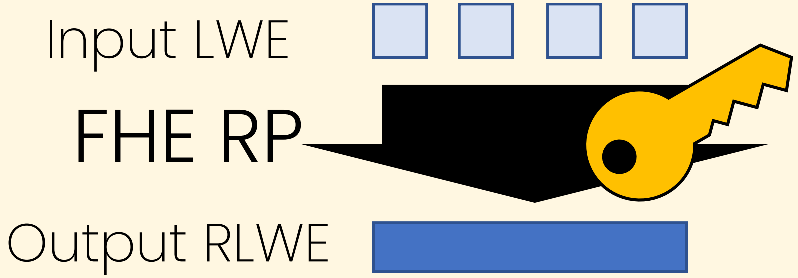

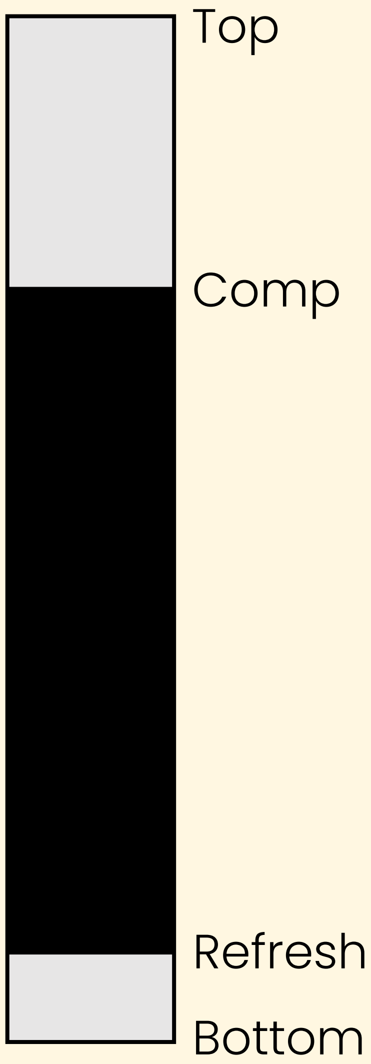

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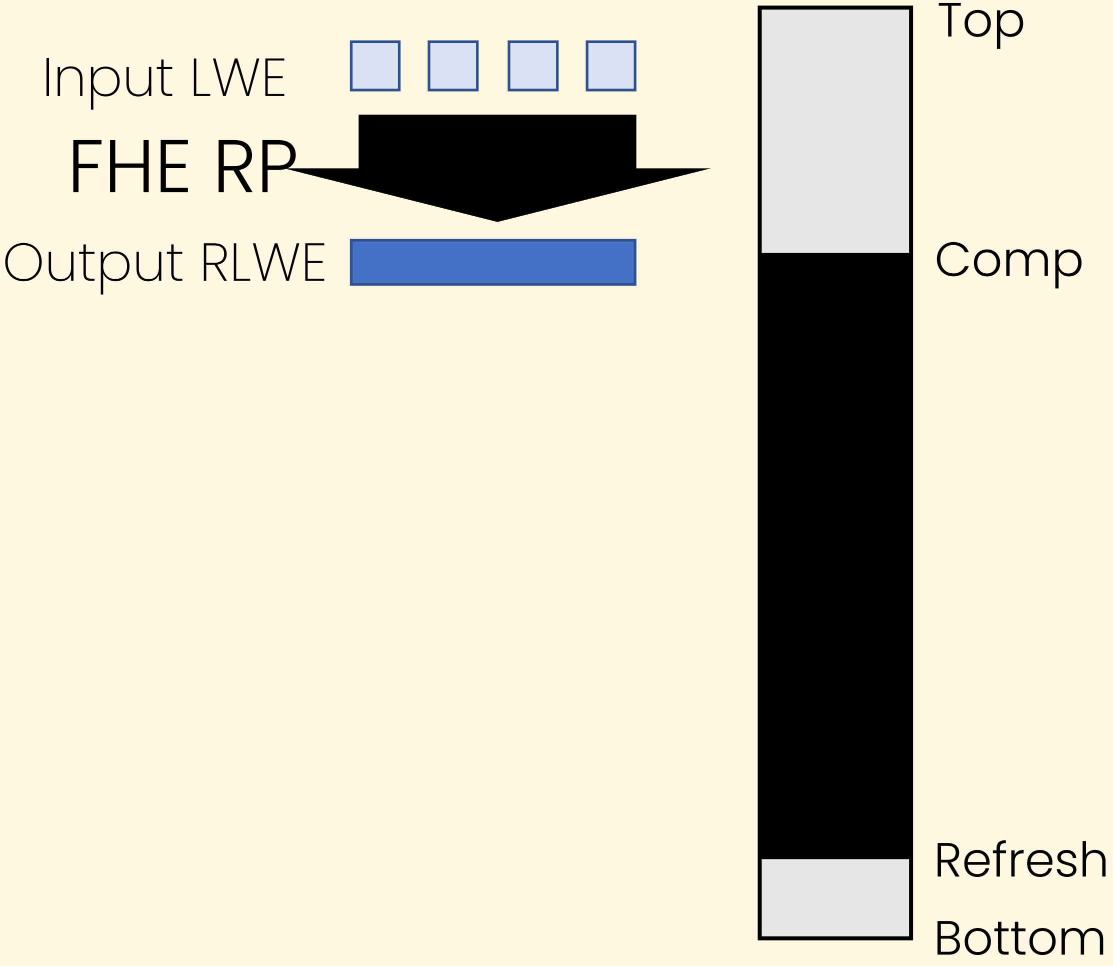
Accelerating FHE RP

RLWE Moduli Optimization

Conventional approach



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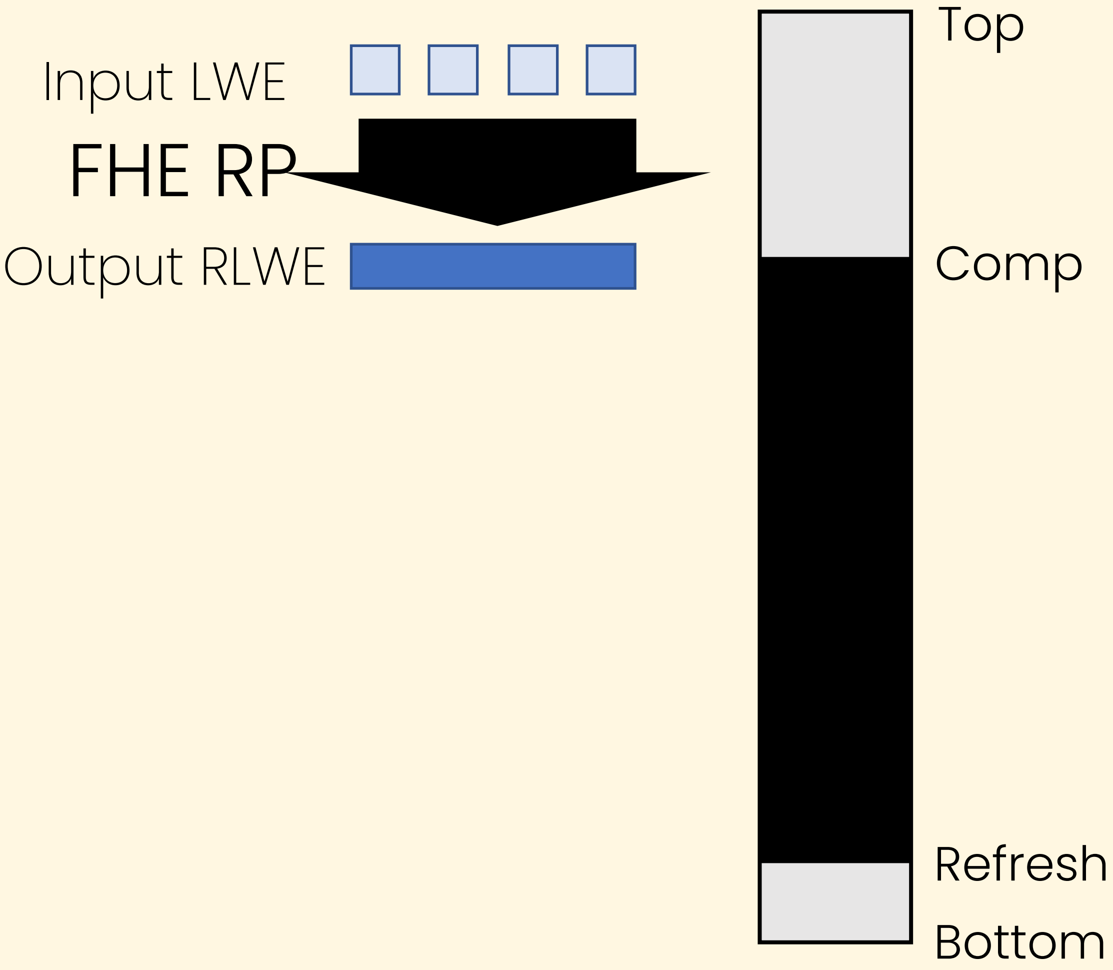

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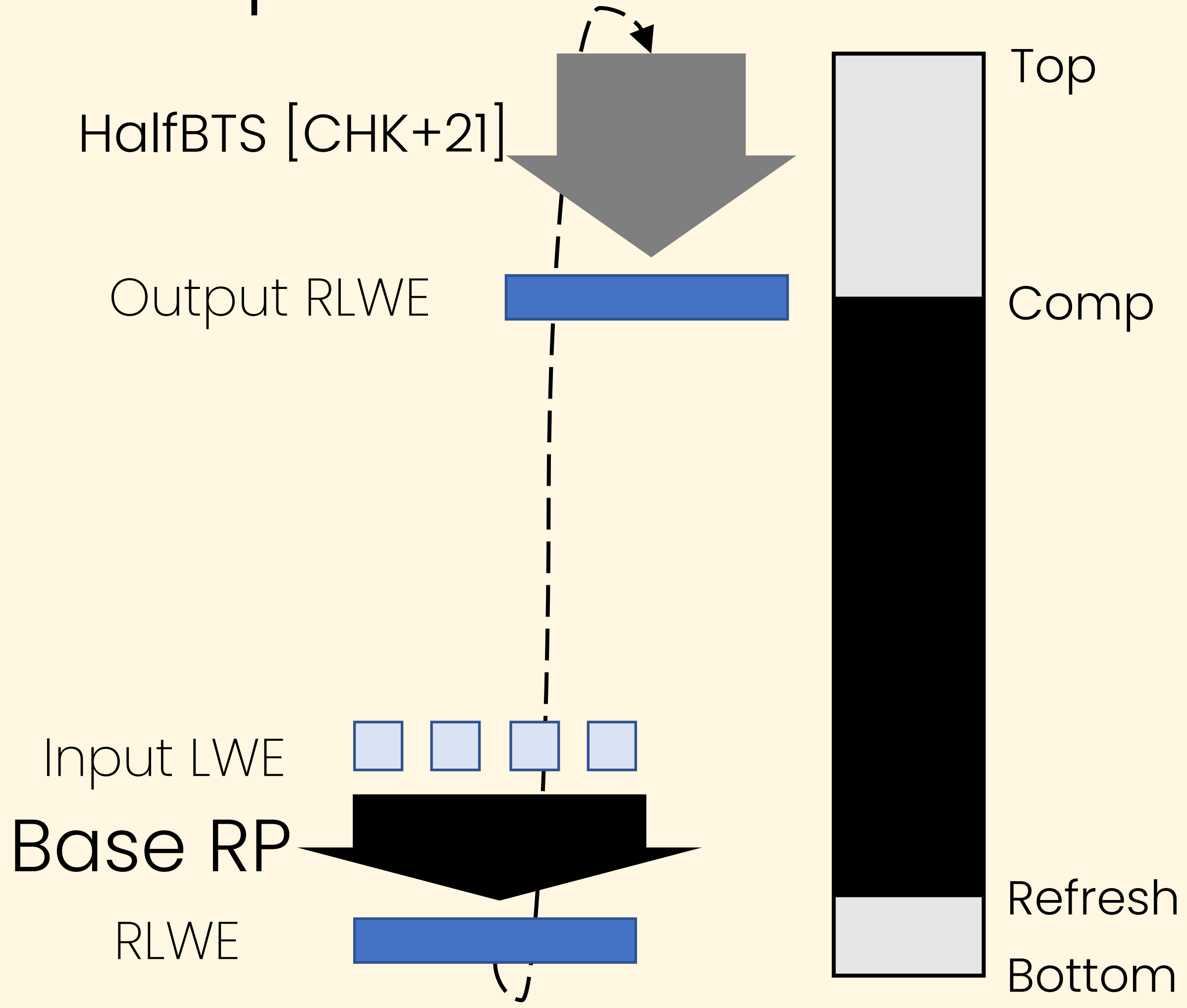

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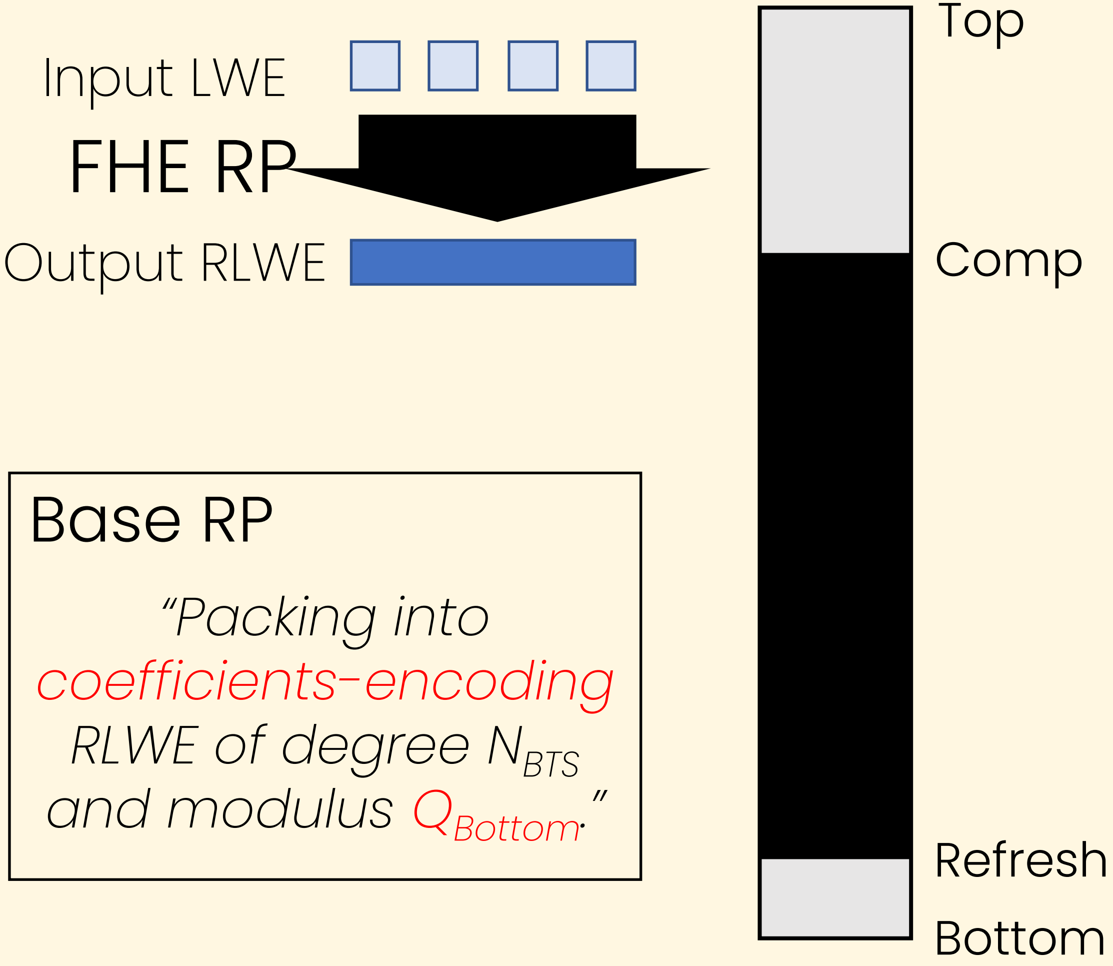

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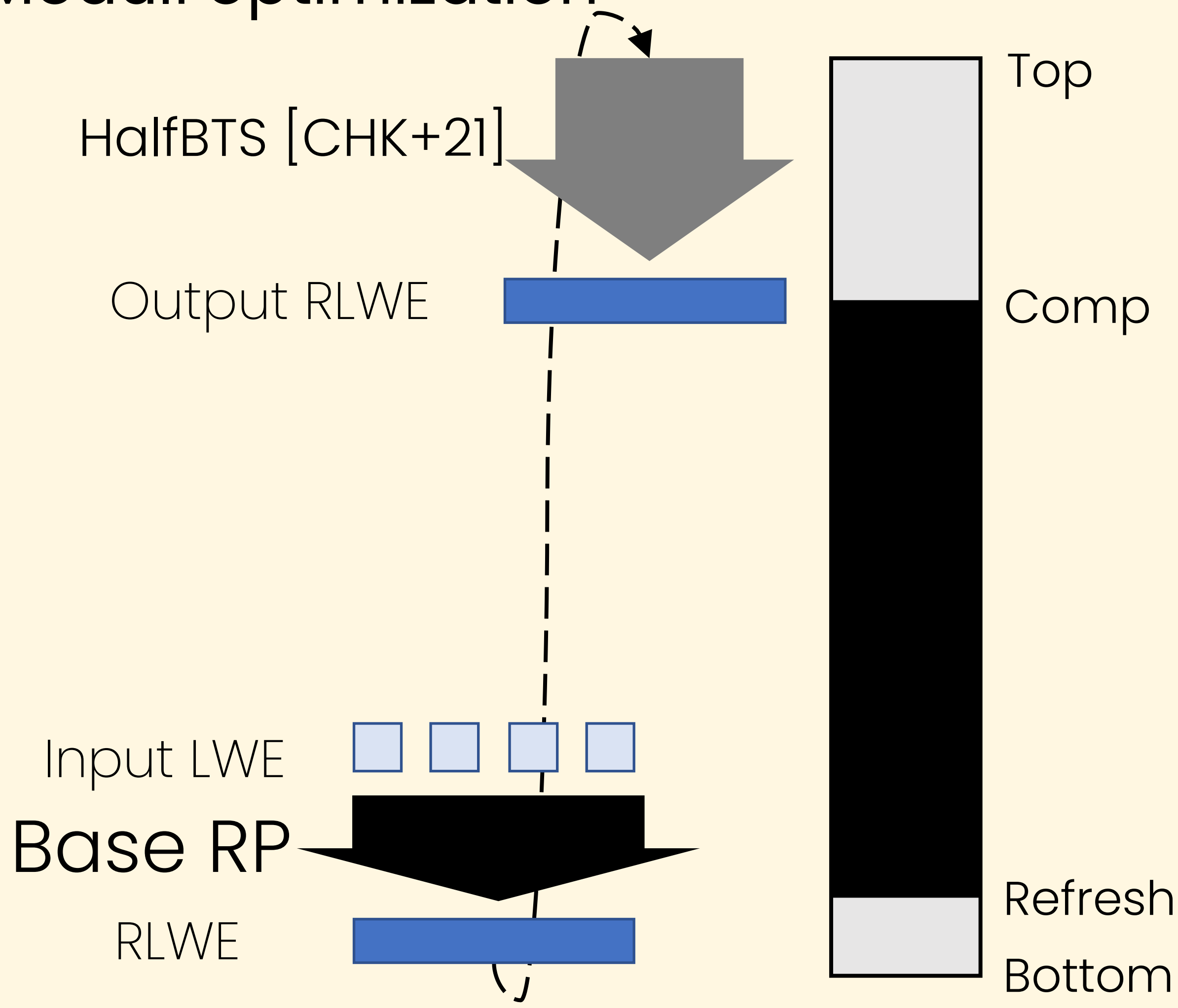
Moduli optimization



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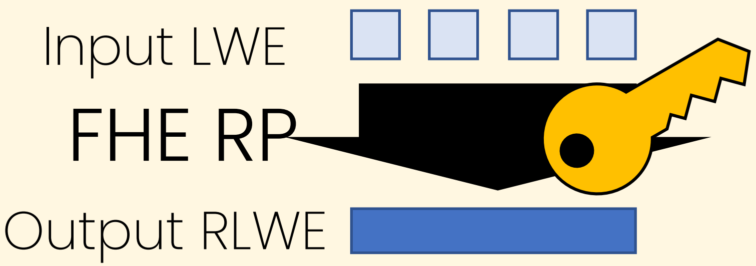

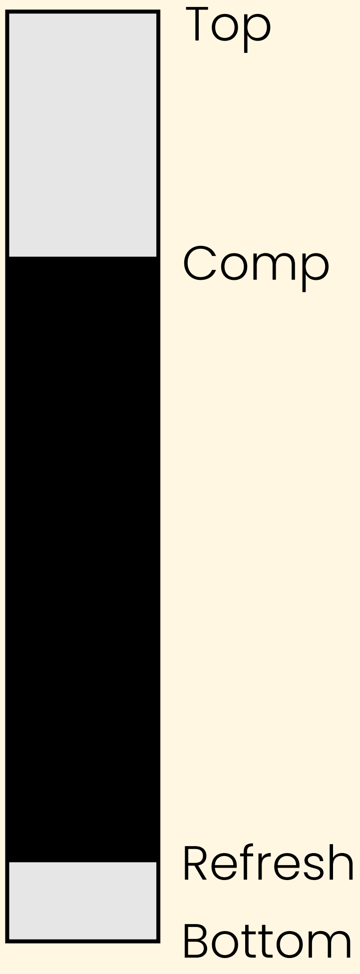

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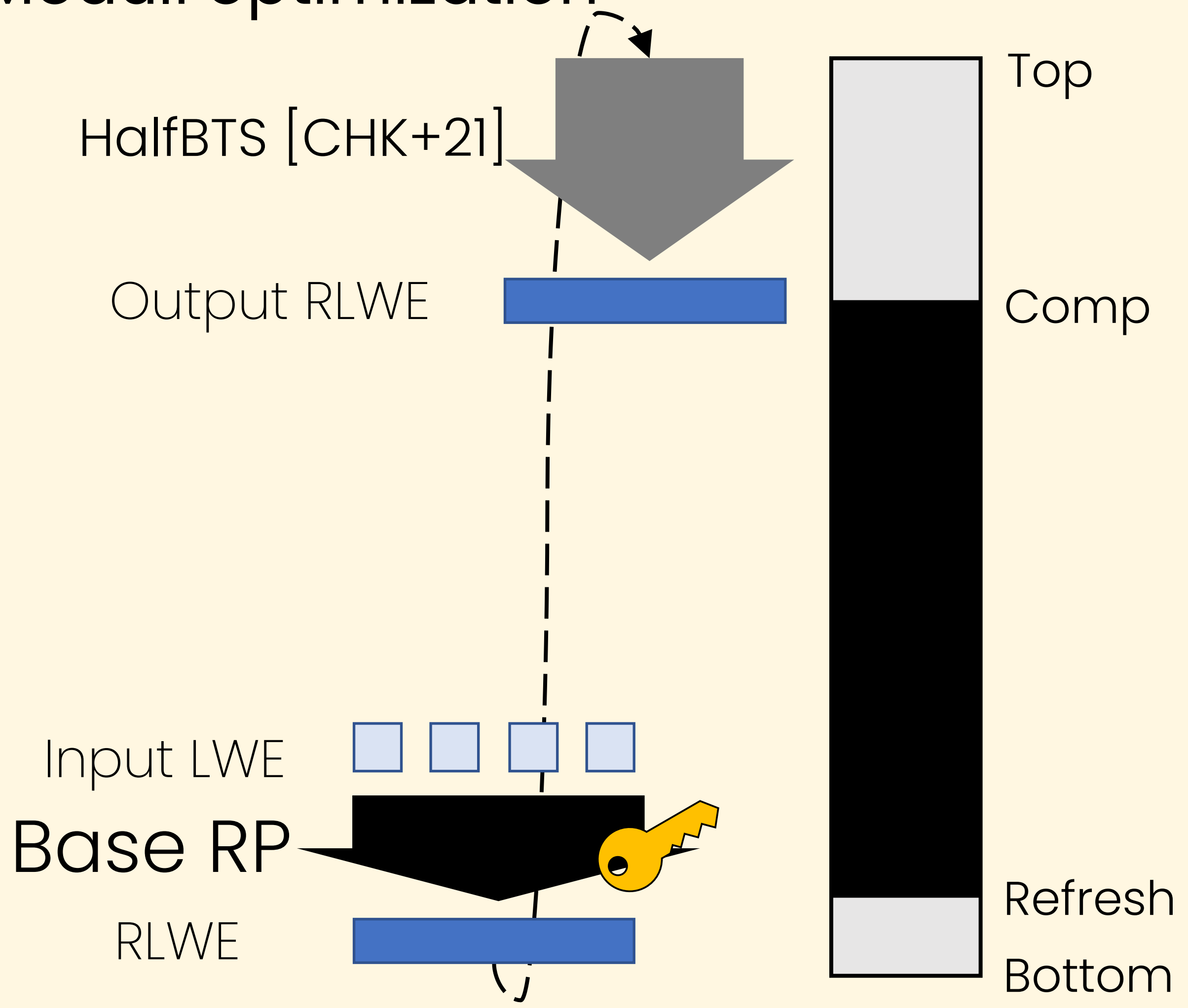

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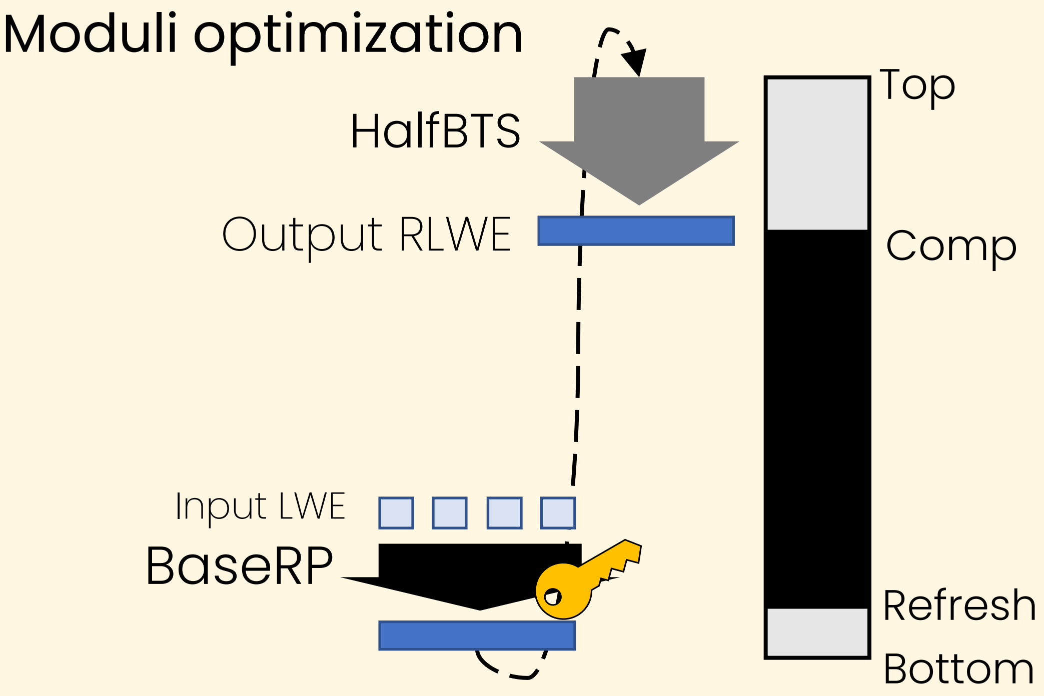

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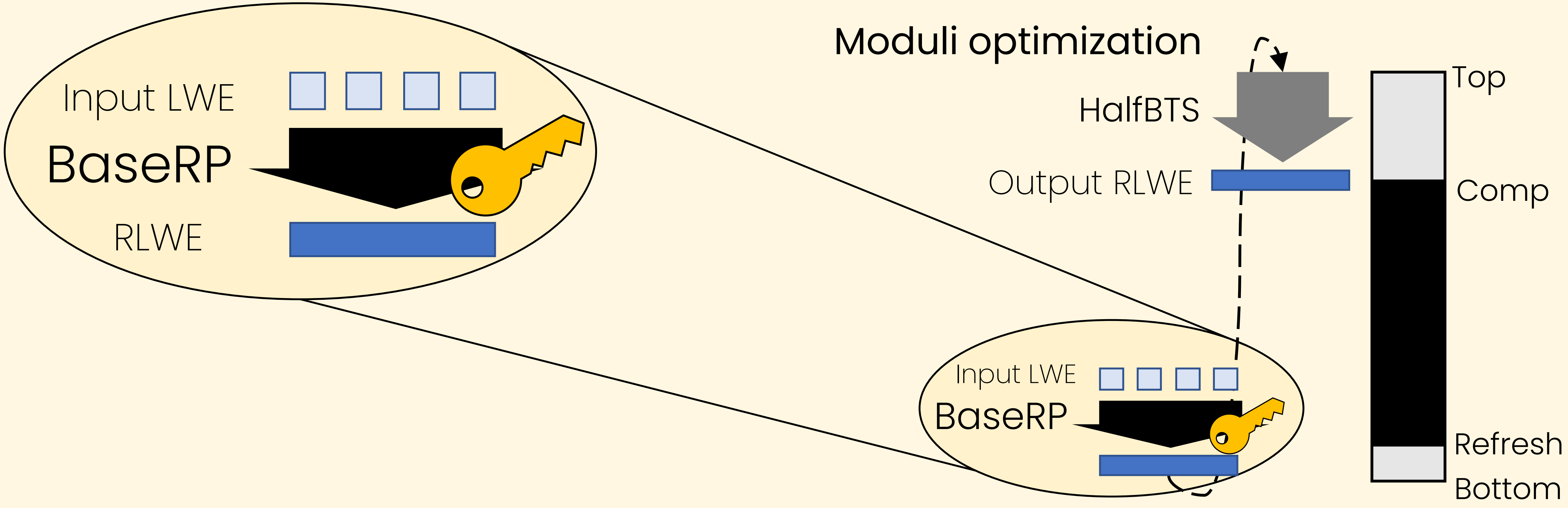

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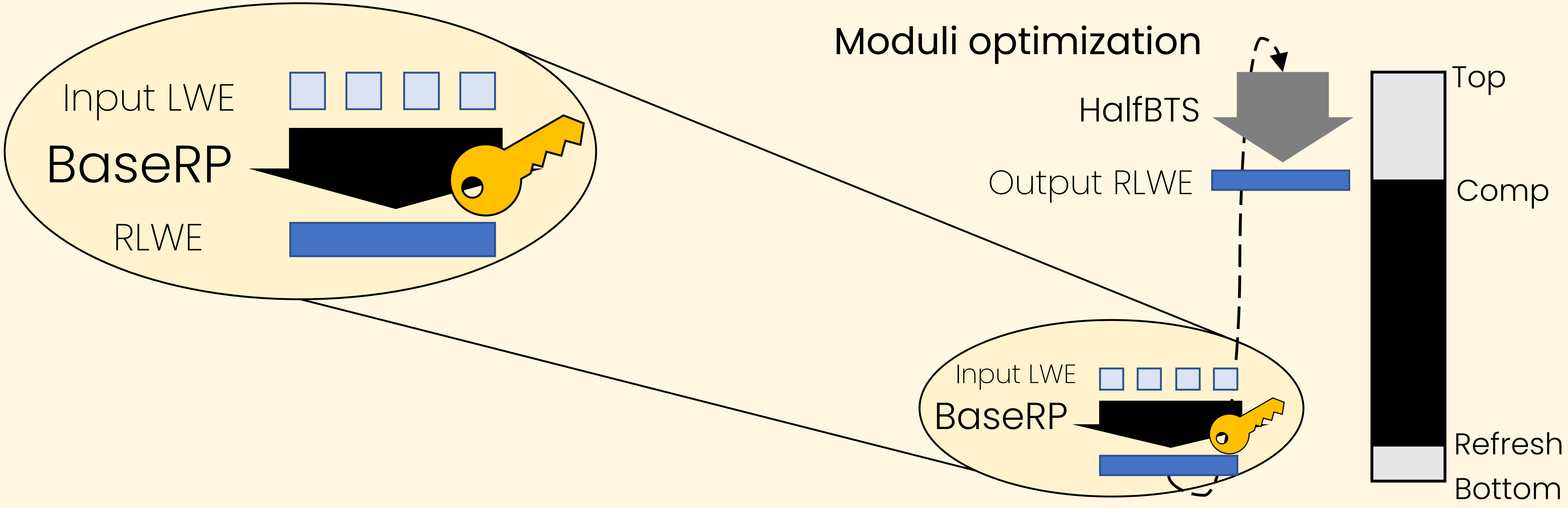

Ring switching (RS) [GHPS13]

- Small moduli allow a small degree RLWE.
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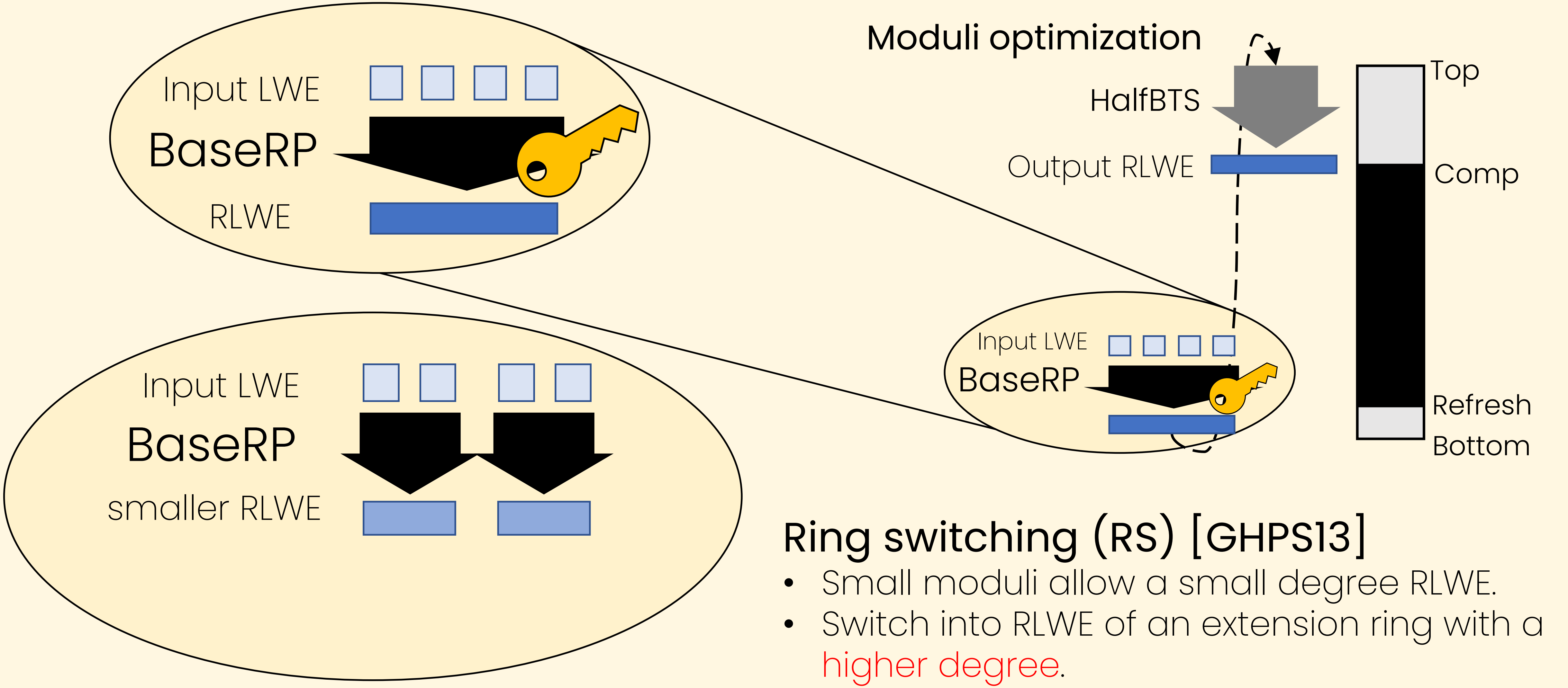

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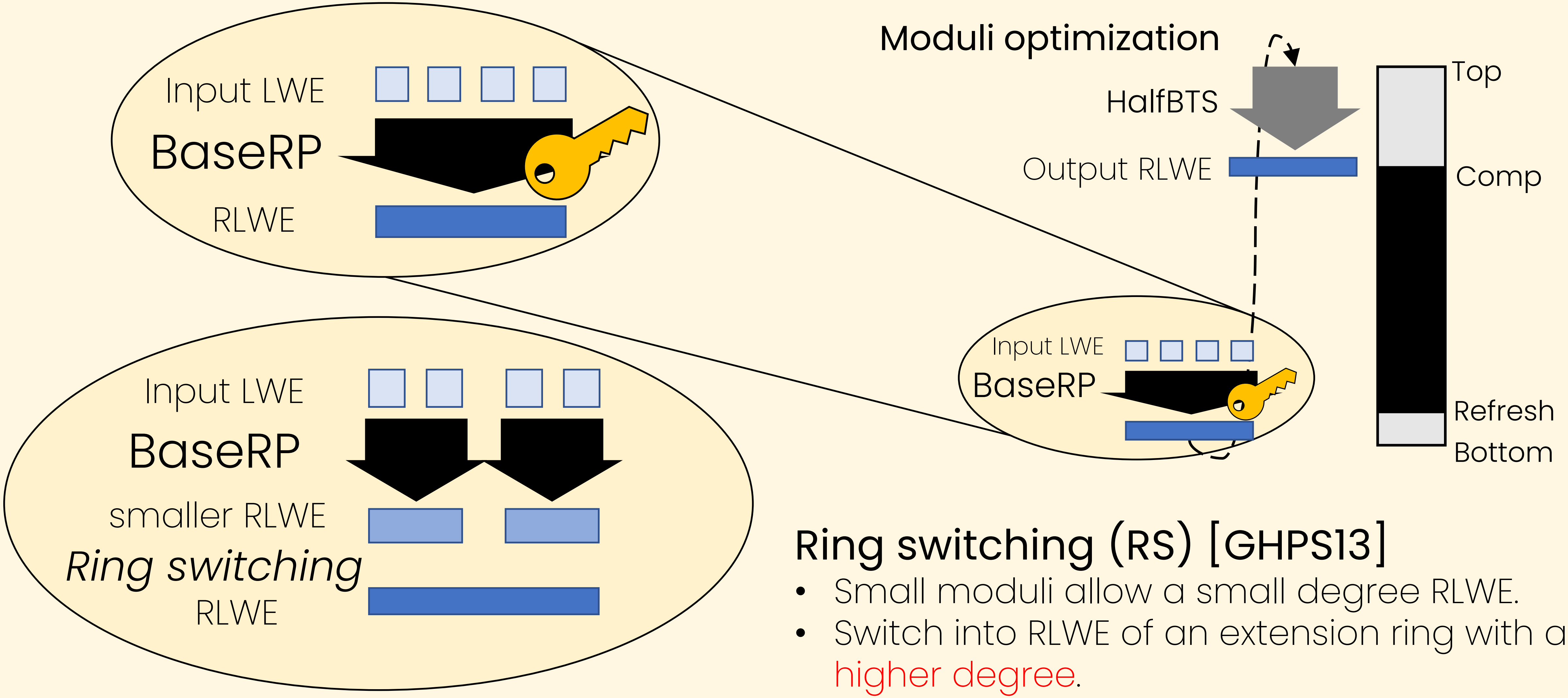

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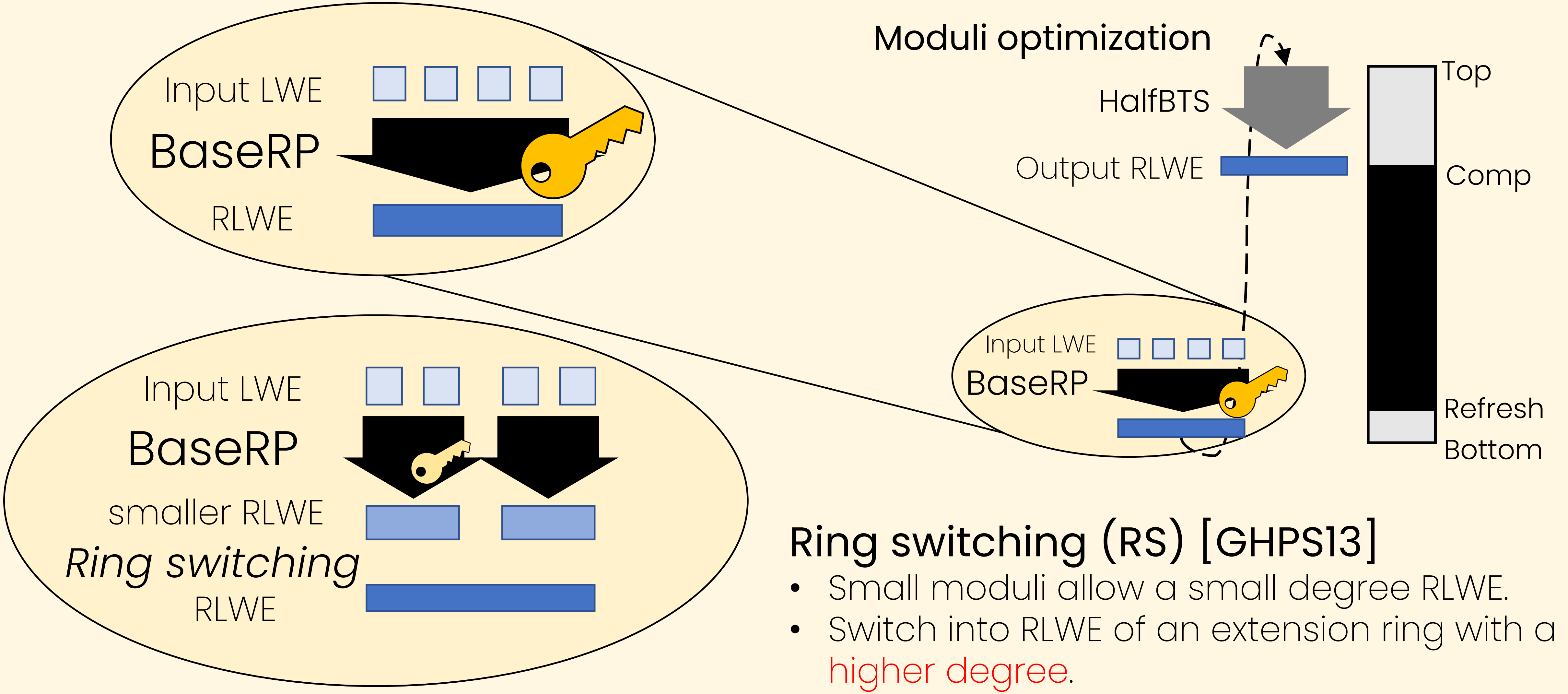

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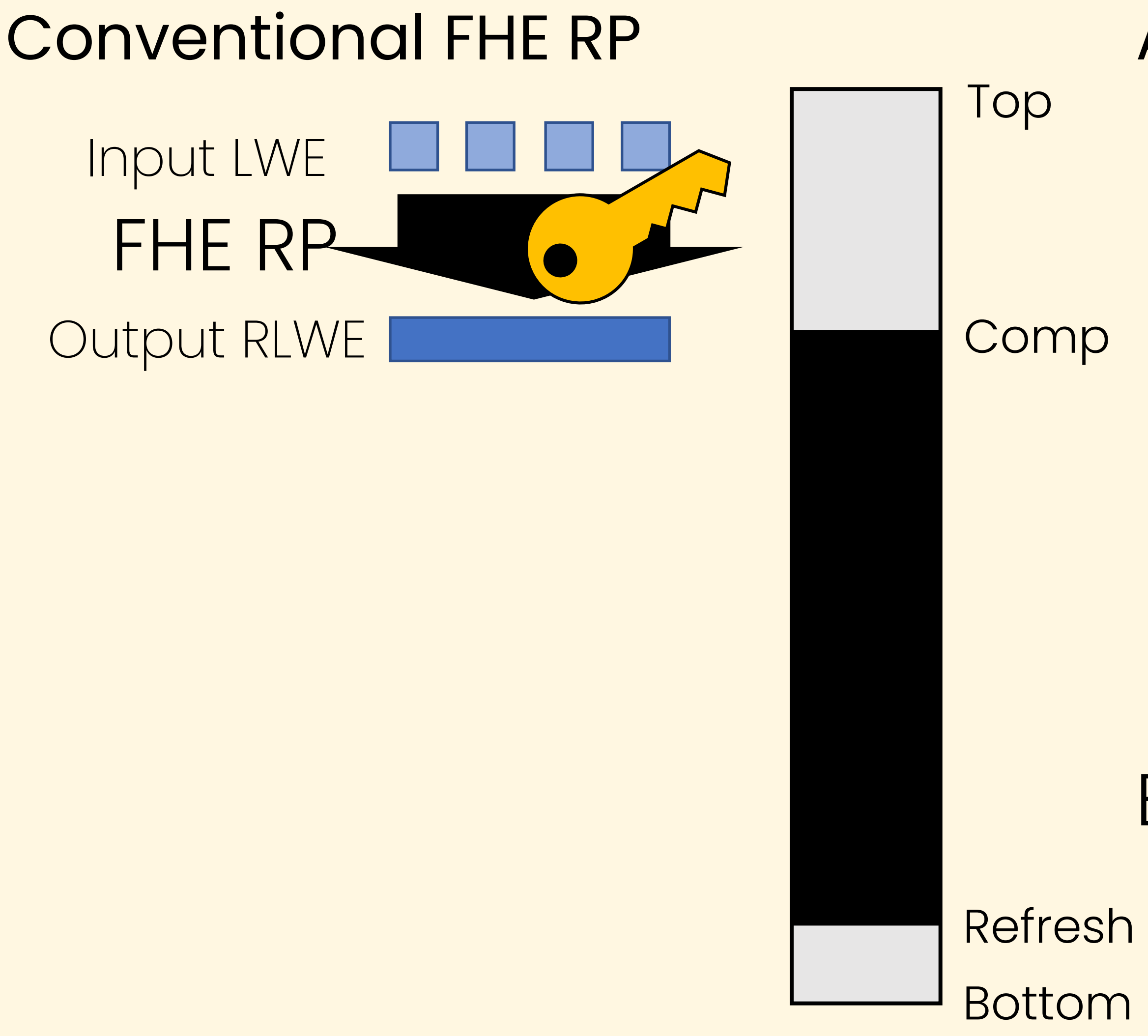
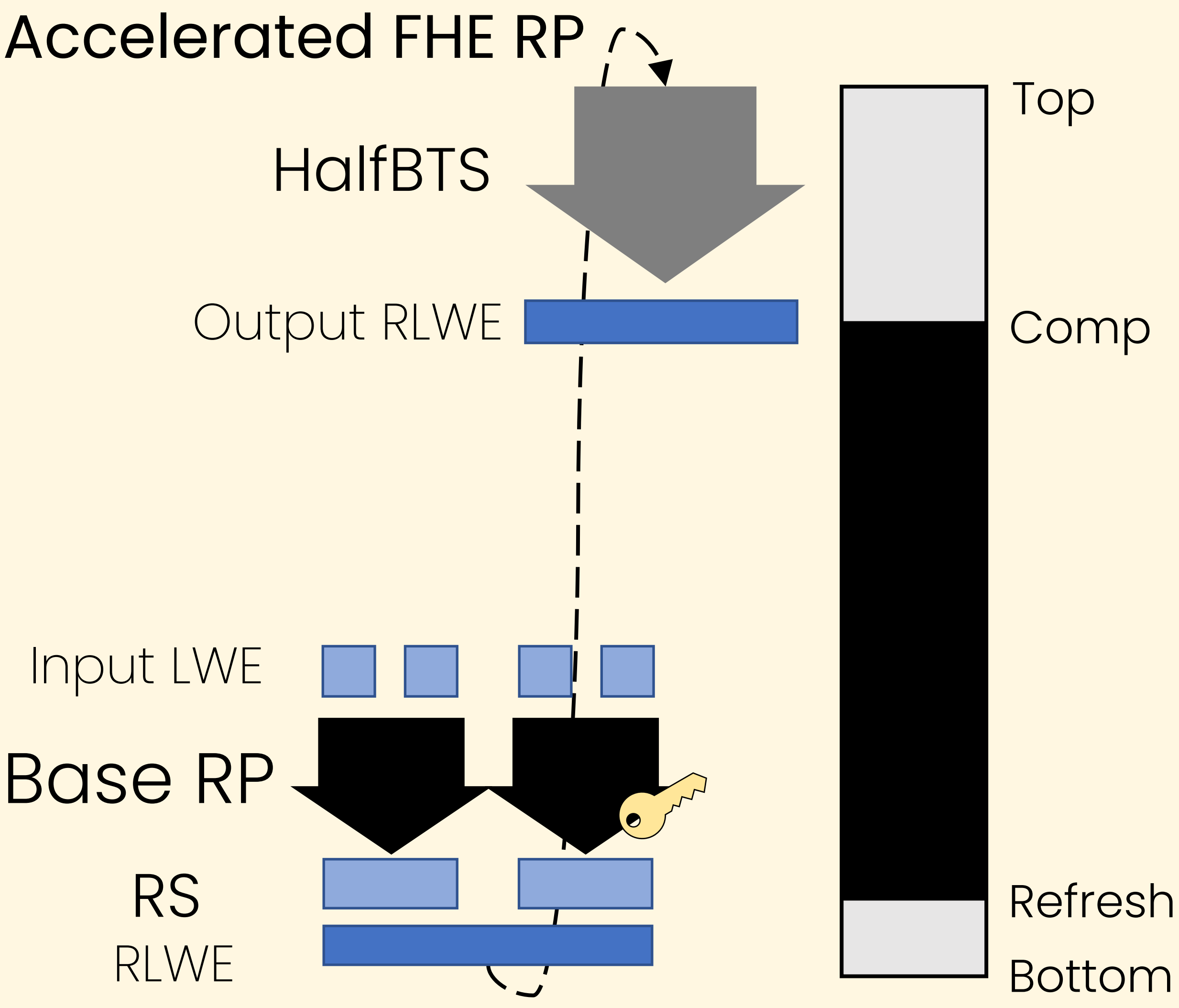
Improved FHE RP





Existing Approaches

RP as a Matrix Multiplication

- LWE ciphertexts $\mathbf{ct}_i$: $c_{i\,1}\,s_1 + c_{i\,2}\,s_2 + \cdots + c_{i\,K}\,s_K \approx m_i$ for each i

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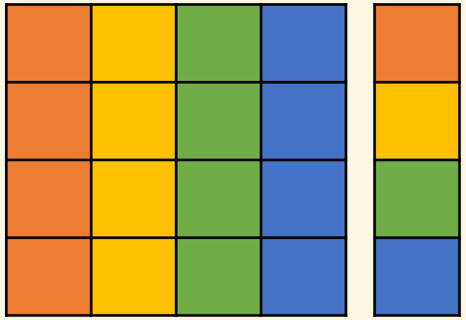
- RP is (plaintext) matrix – (ciphertext) vector multiplication in RLWE formats.

Existing Approaches

- Three approaches to encode the plaintext matrix.

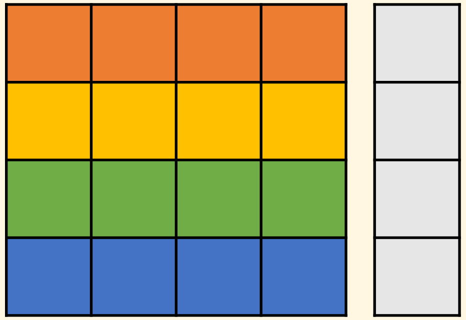
Column method

- CGGI17, BGGJ20
- K key switchings
- K keys
- Consumes $\mathbf{0}$ level



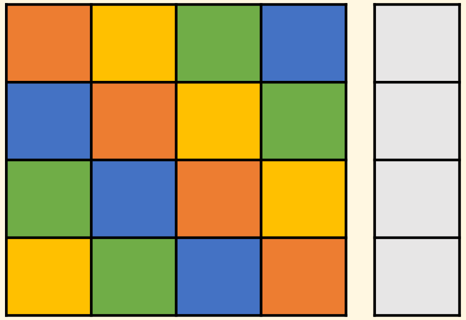
Row method

- CDKS21
- K key switchings
- $\log K$ keys
- Consumes $\mathbf{1}$ level

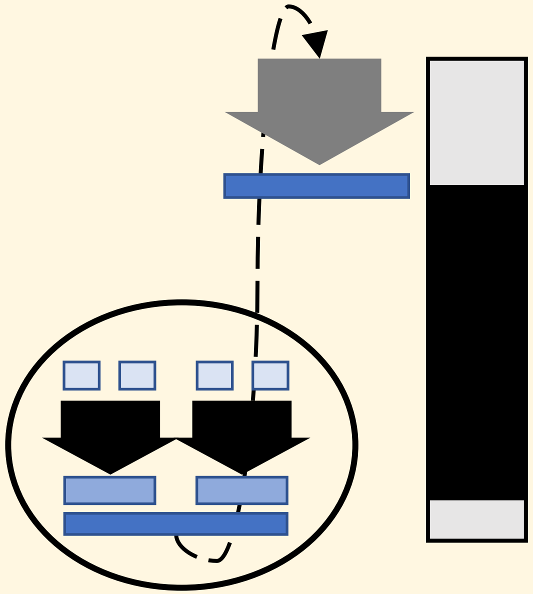


Diagonal method

- HS14, LHH+21
- $2\sqrt{K}$ key switchings
- $2\sqrt{K}$ keys
- Consumes ≥ 4 levels



Base RP with Existing Approaches



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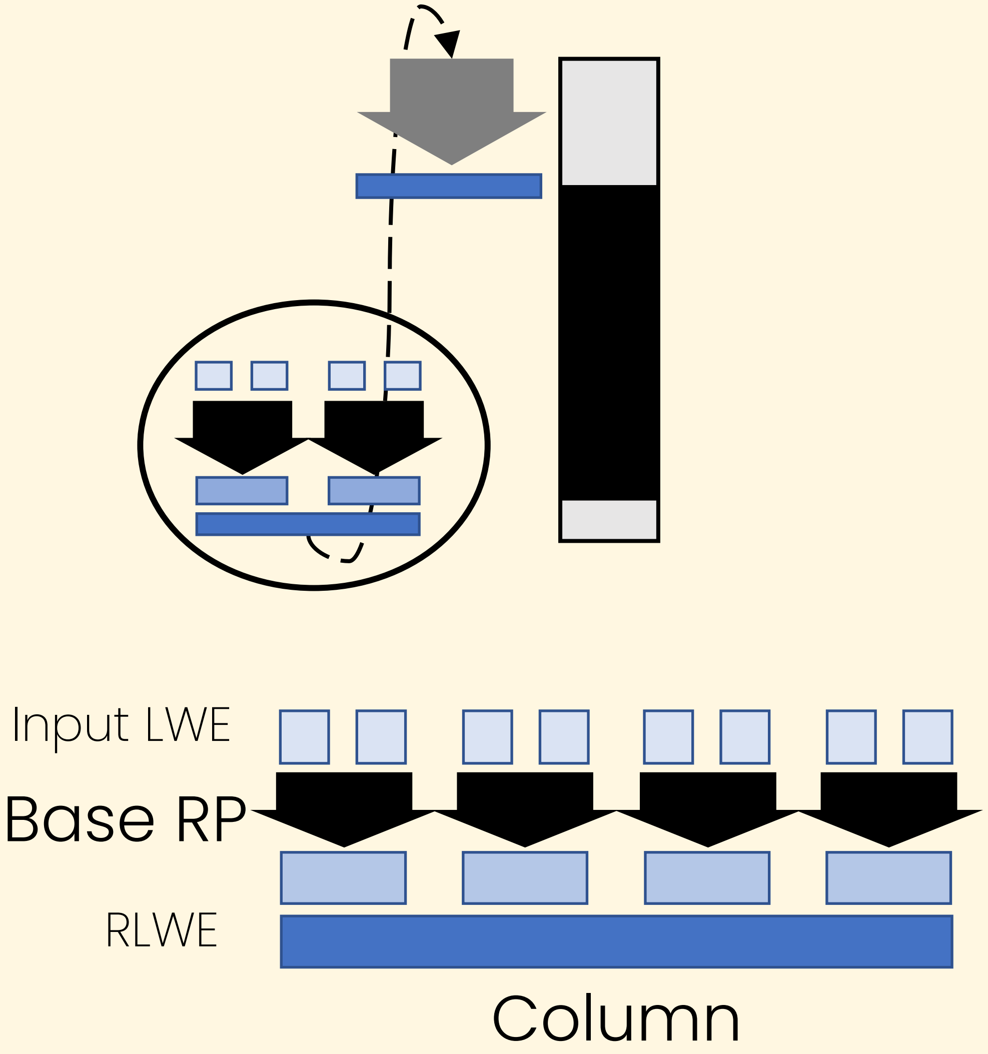

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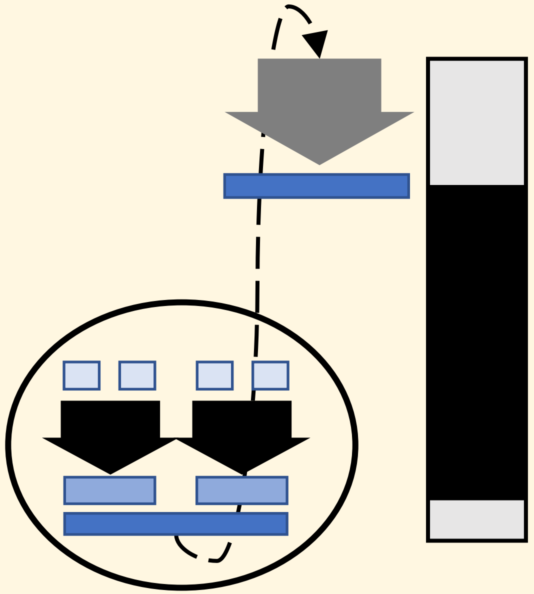

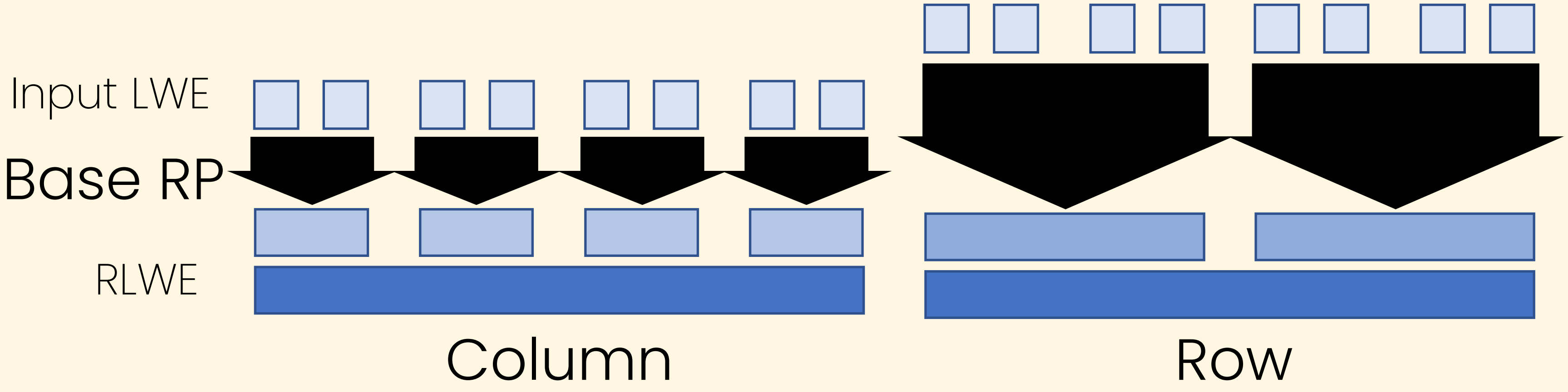

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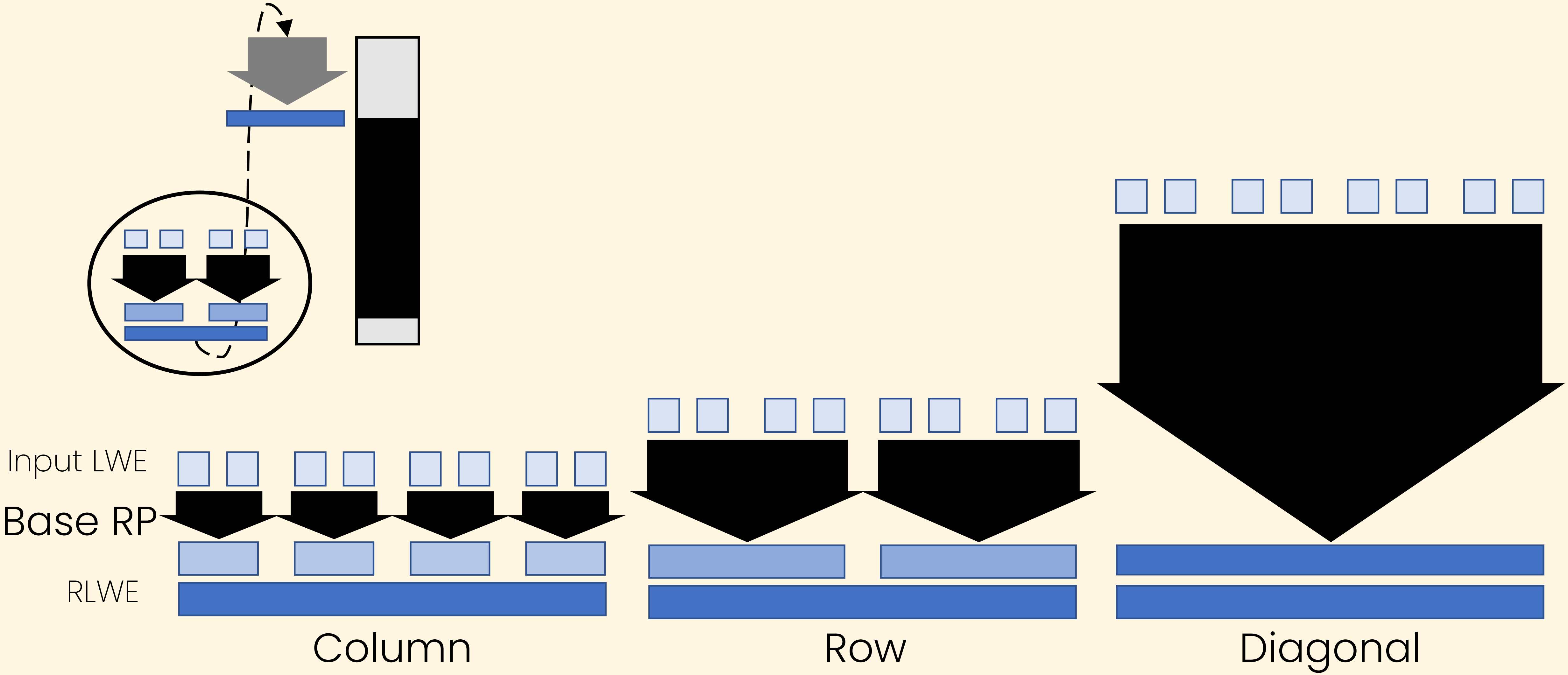

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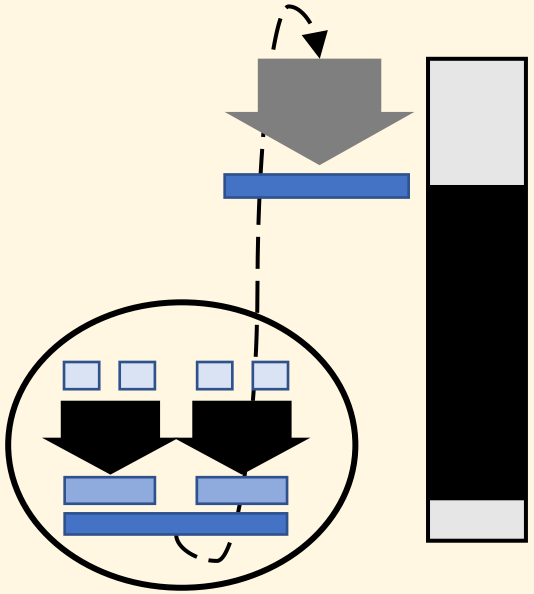

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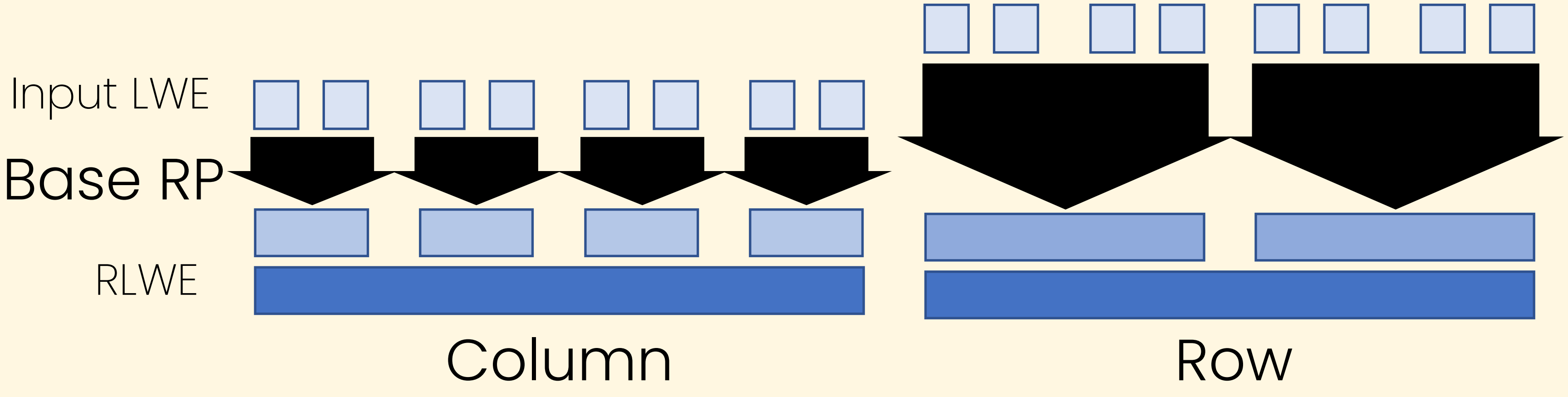

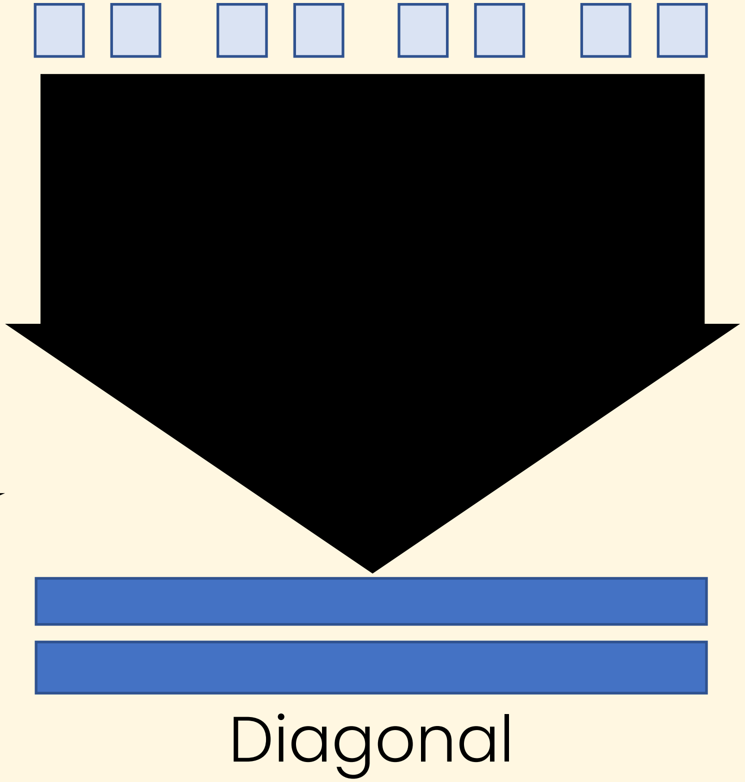

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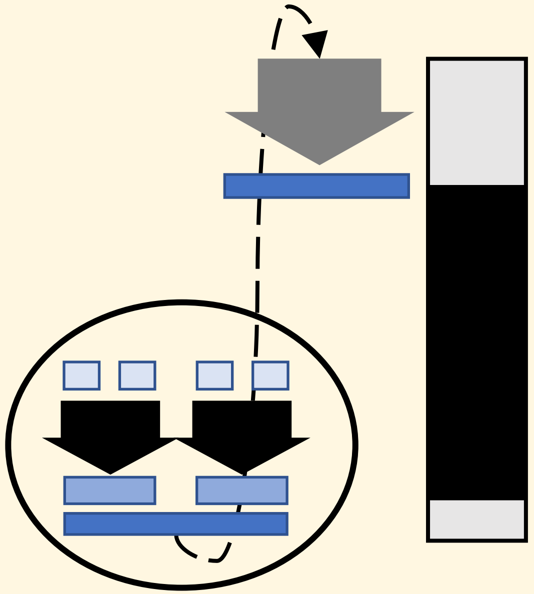

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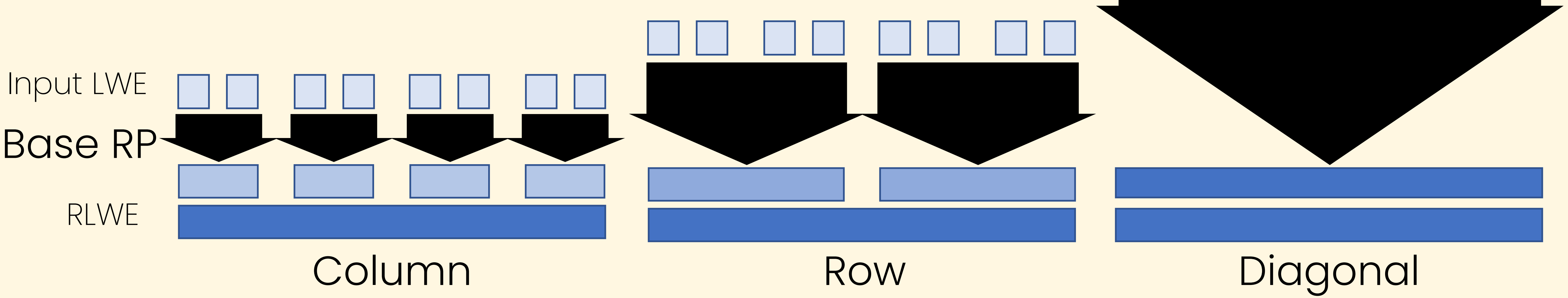

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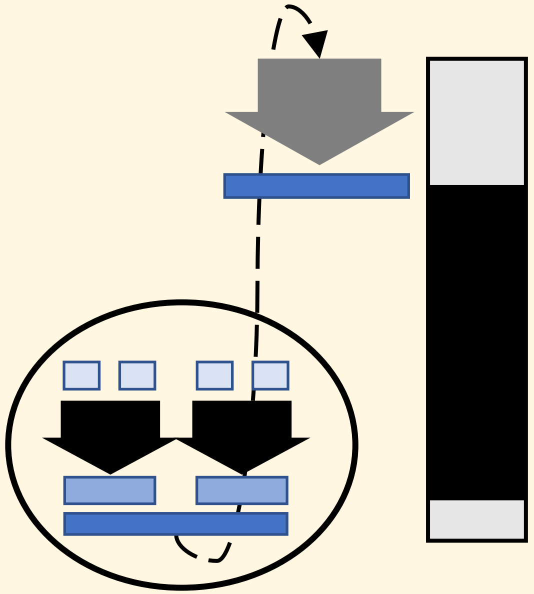

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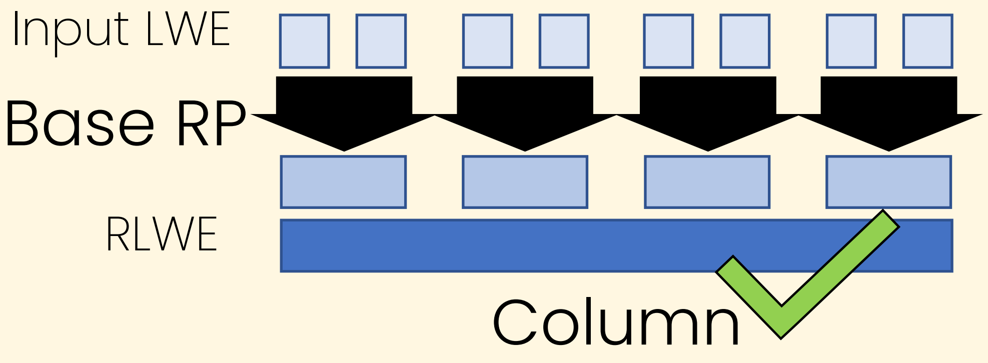
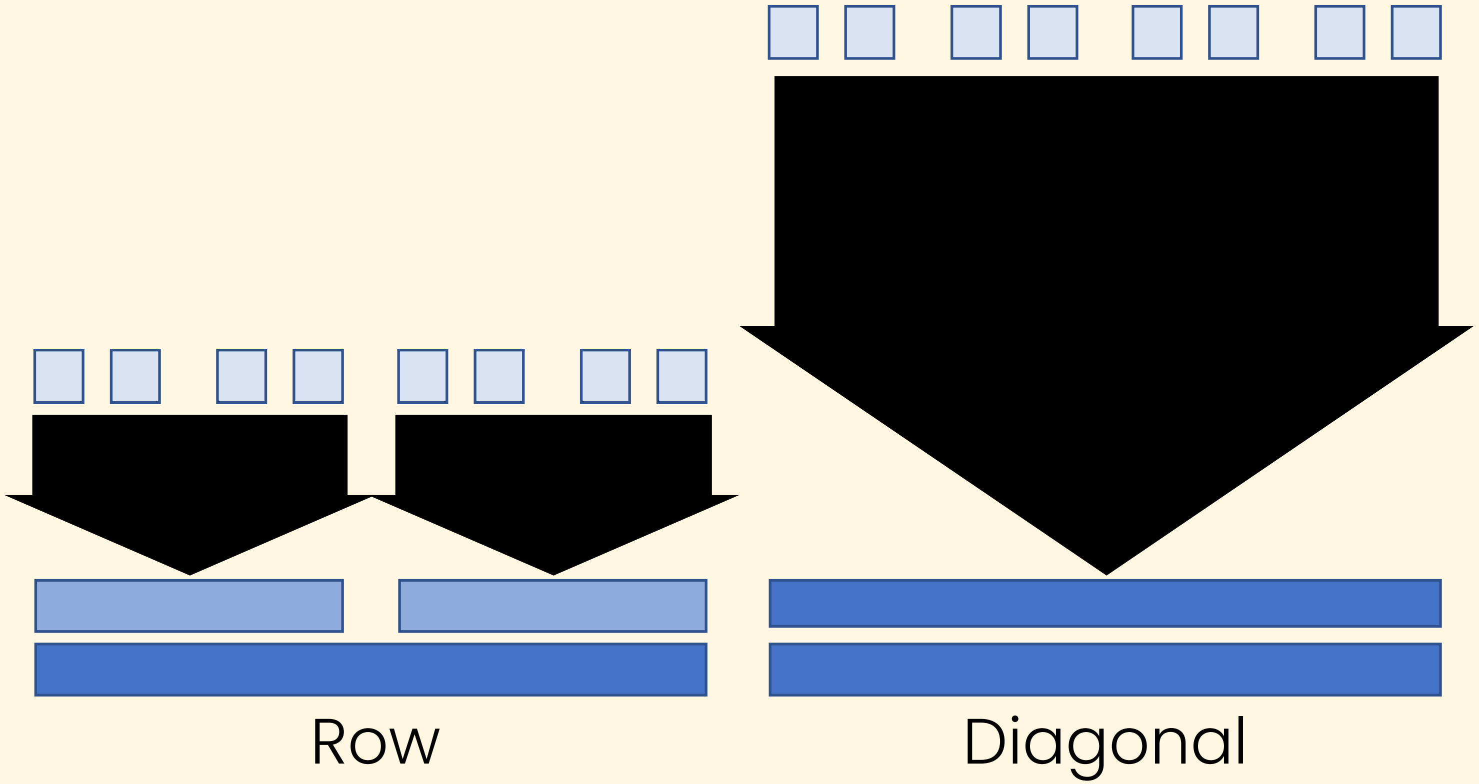
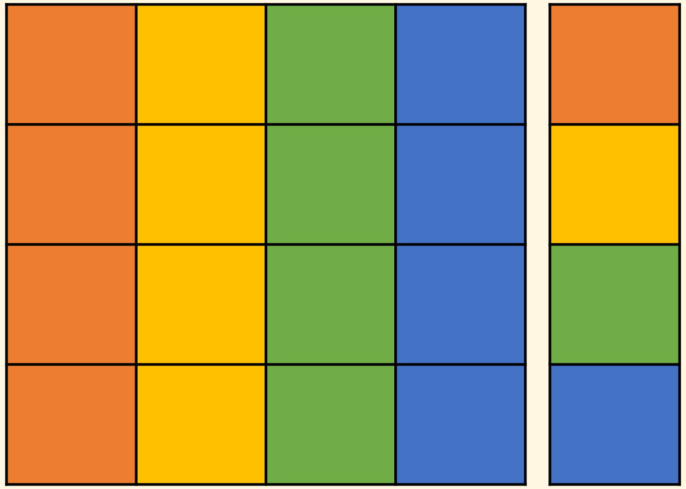




HERMES

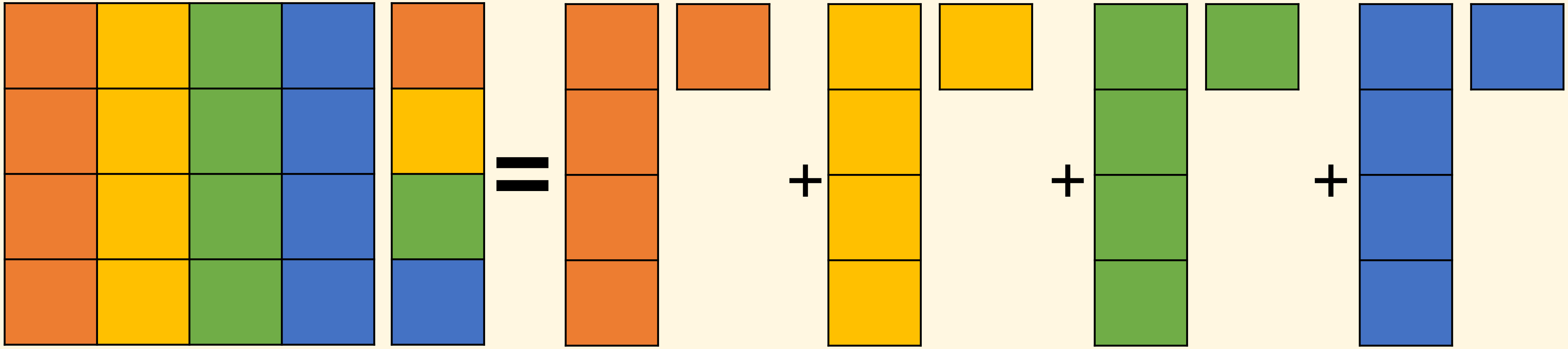
HERMES^0: Column method

HERMES^0: the column method with our optimizations



HERMES^0: Column method

HERMES^0: the column method with our optimizations



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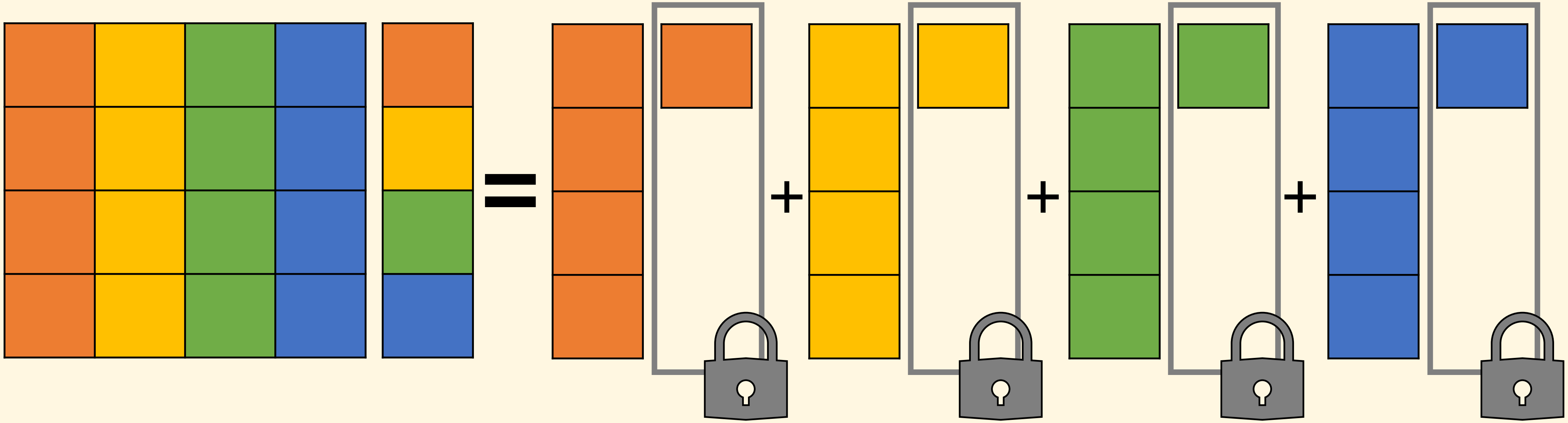

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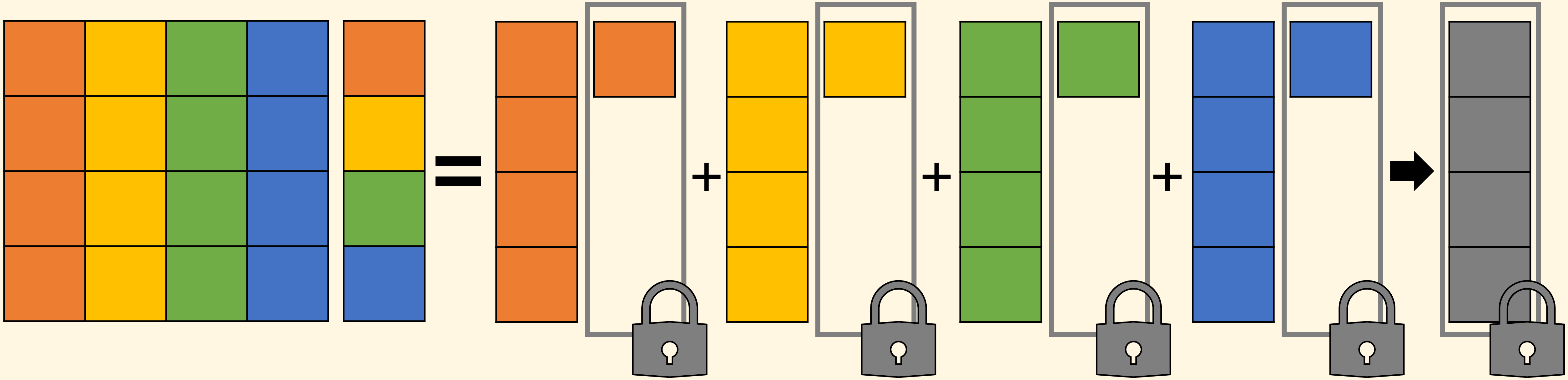

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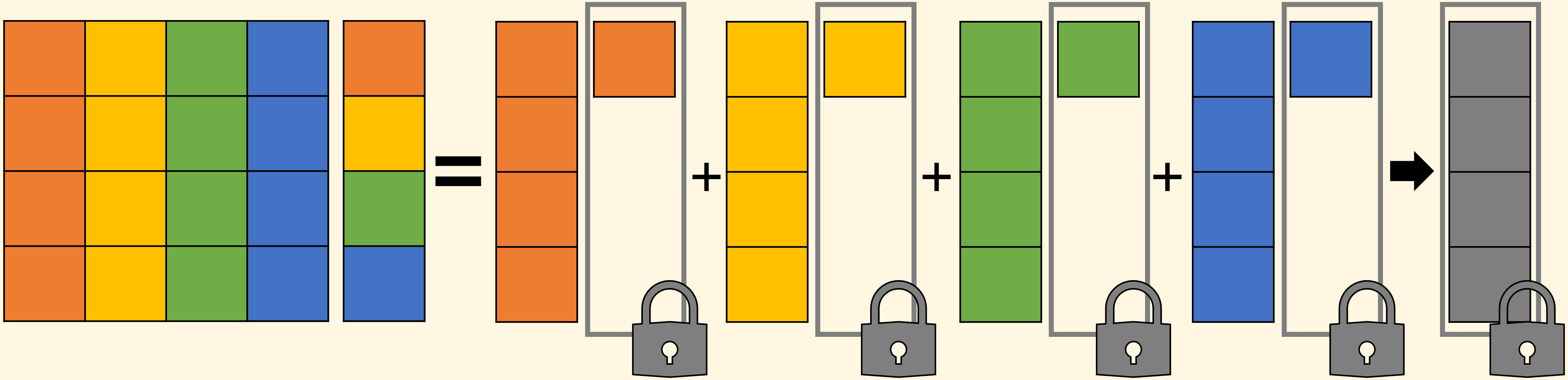
- Practically fast.



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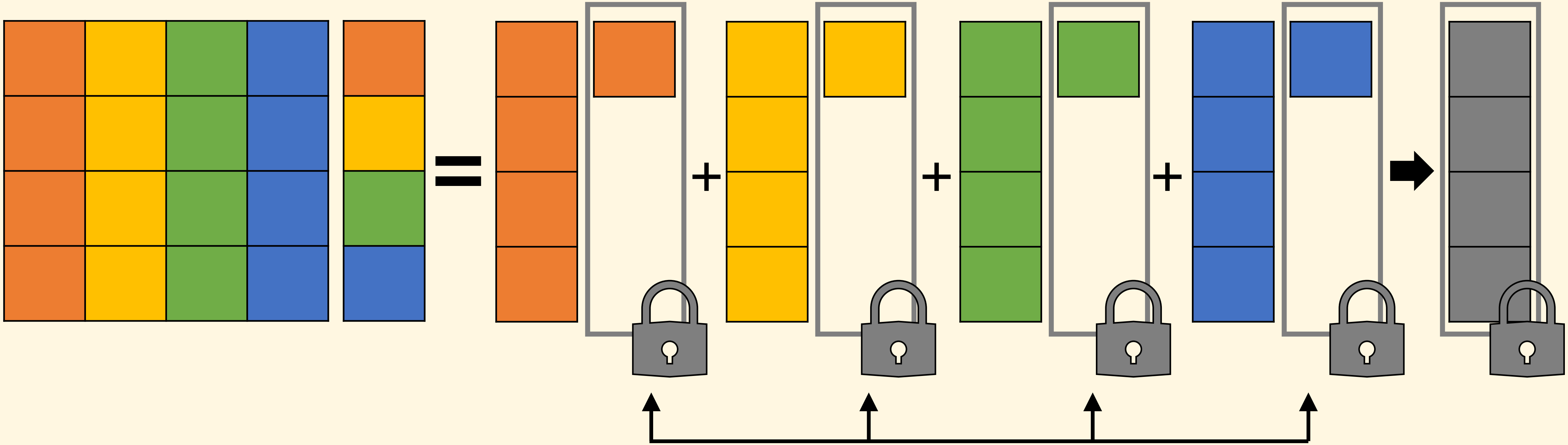

(Switching) Keys for BaseRP $= K$ switching keys

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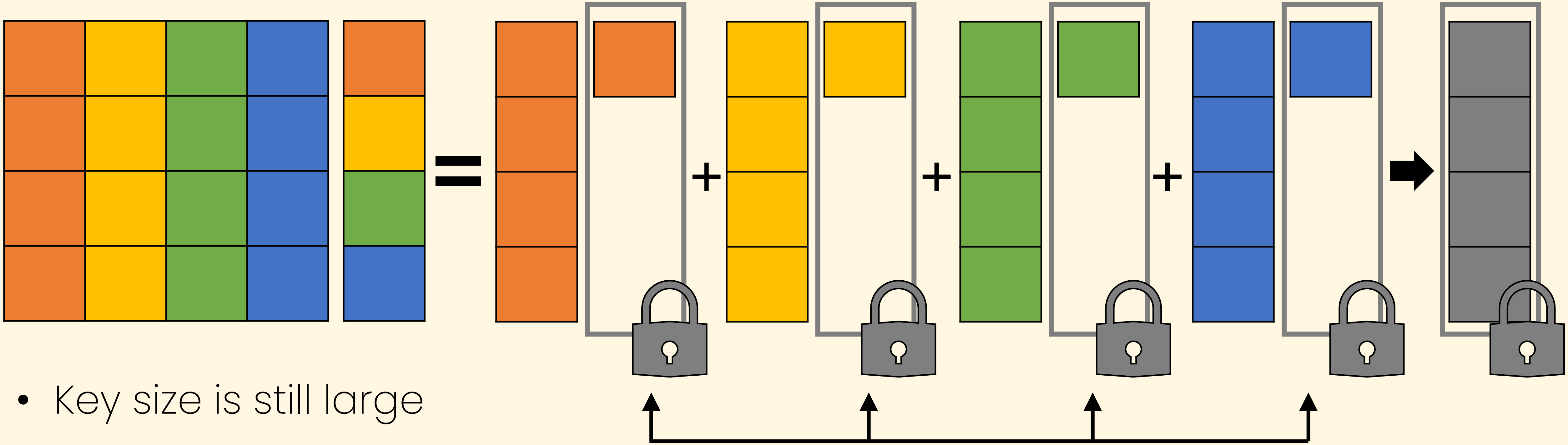

- Key size is still large

(Switching) Keys for BaseRP $= K$ switching keys

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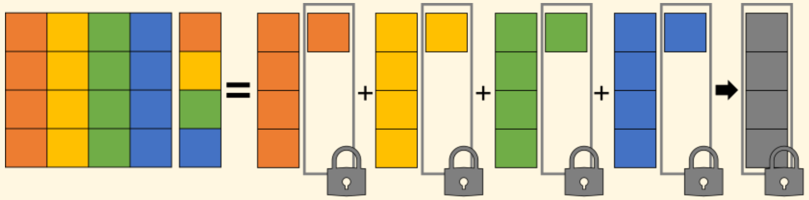

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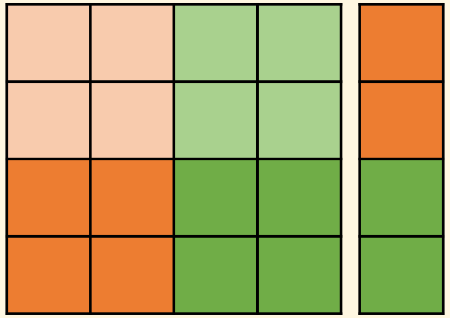

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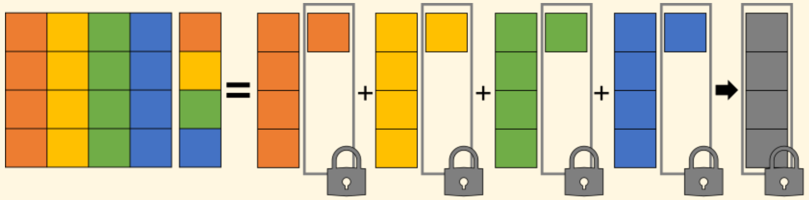

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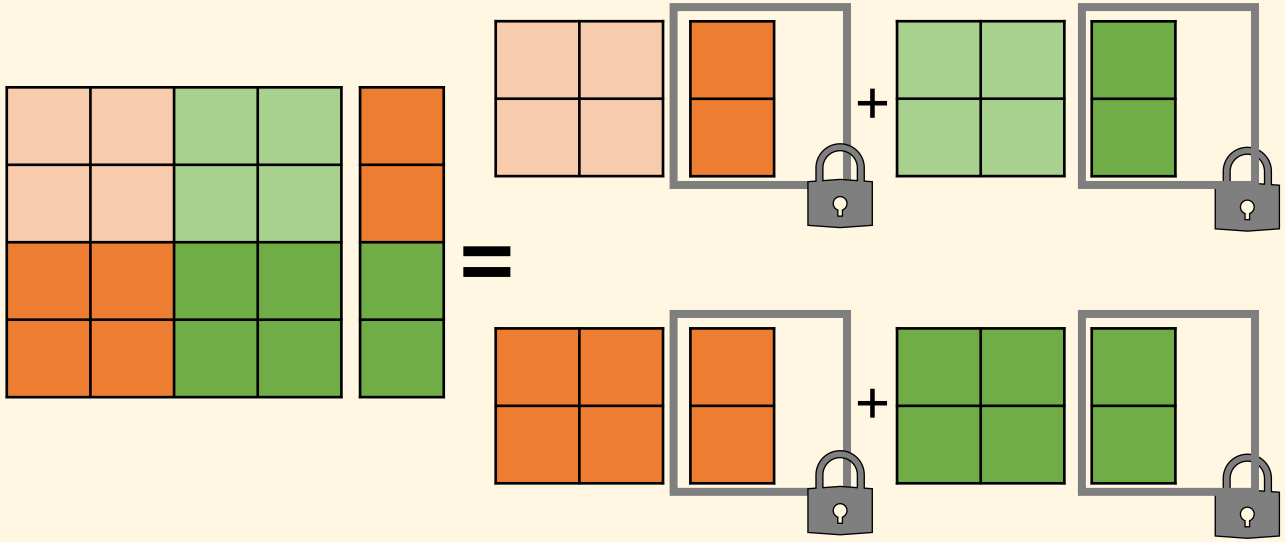

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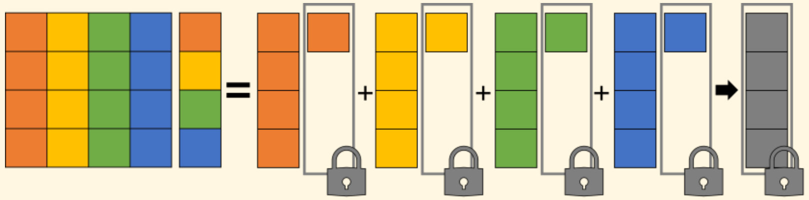

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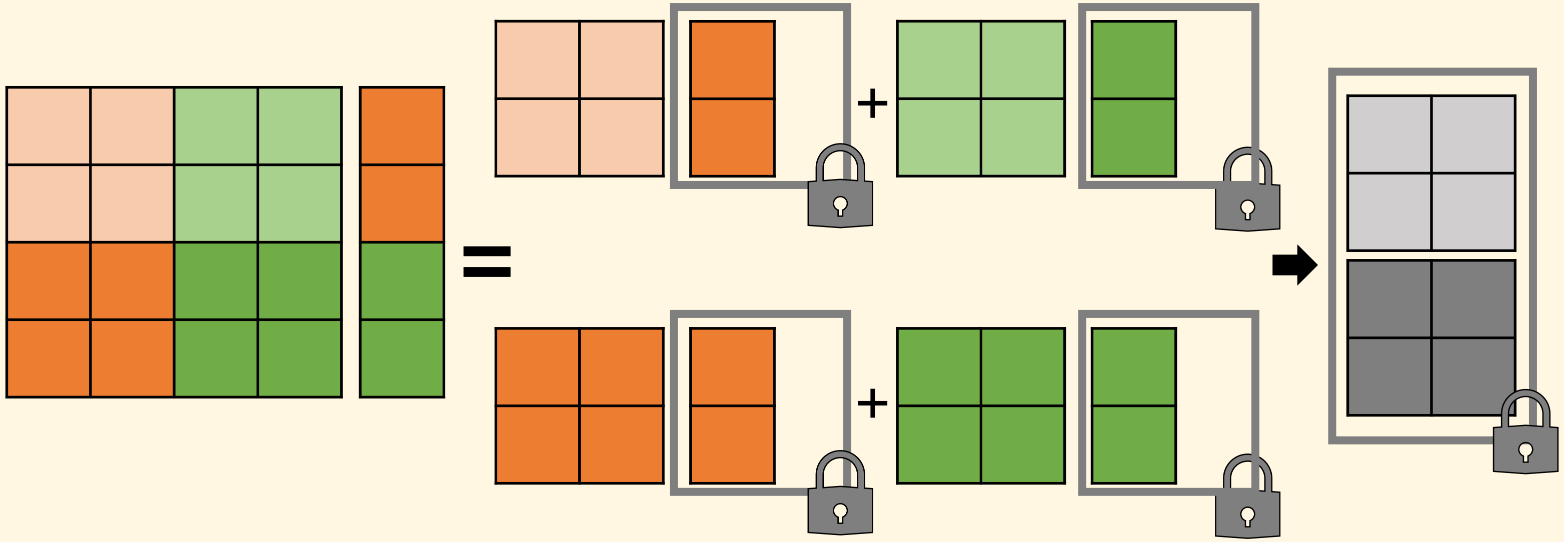

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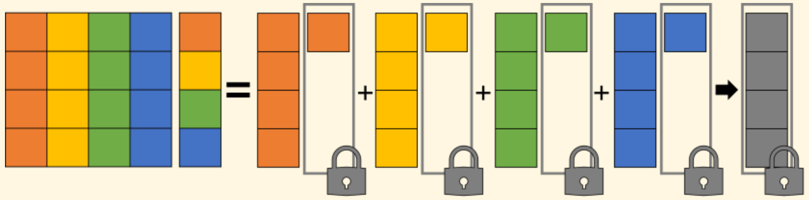

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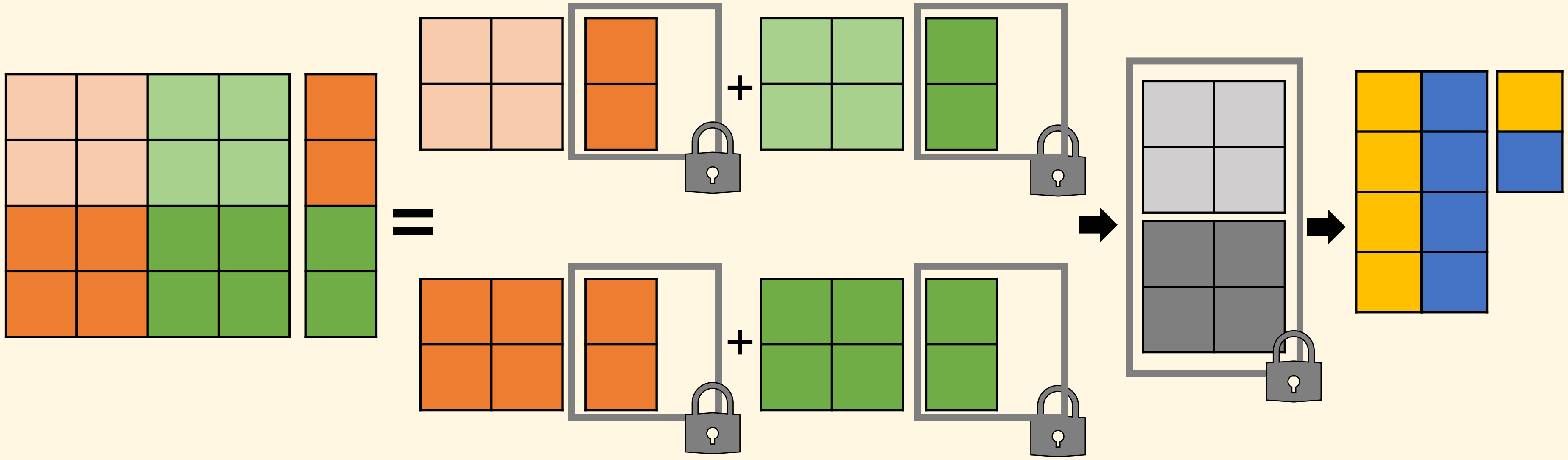

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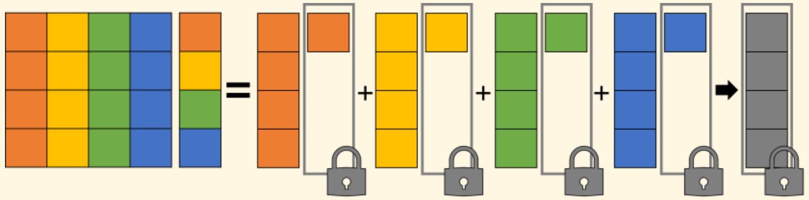

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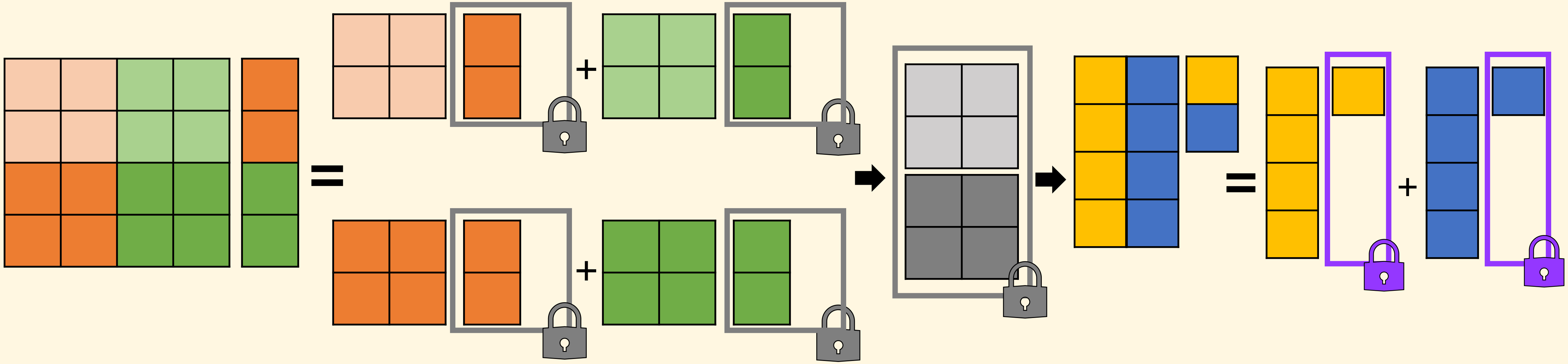

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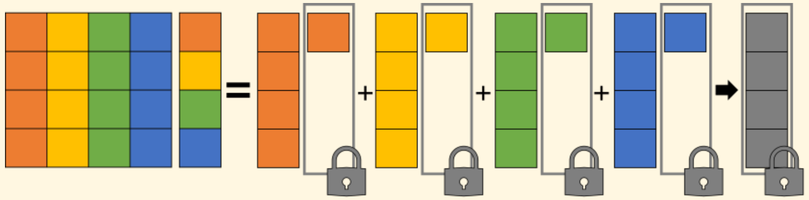

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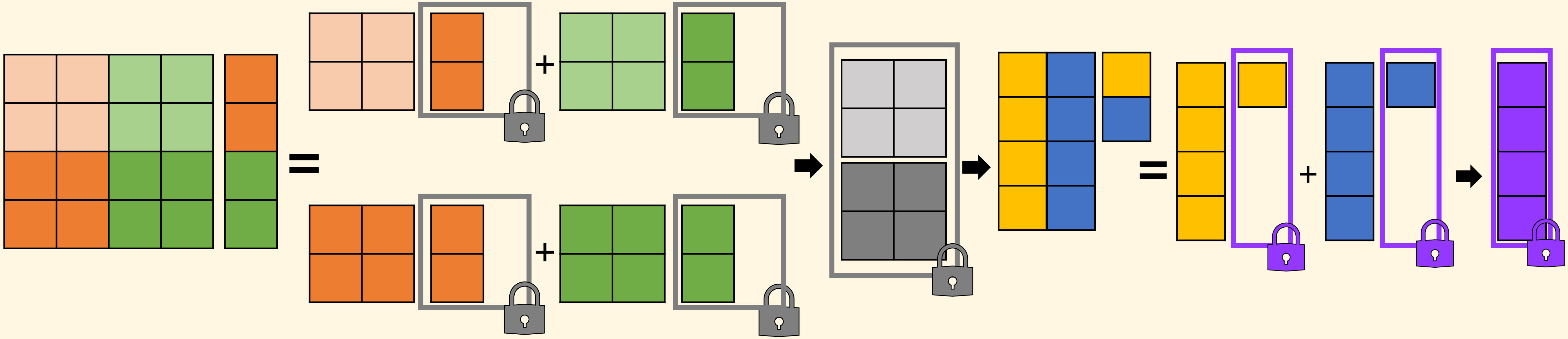

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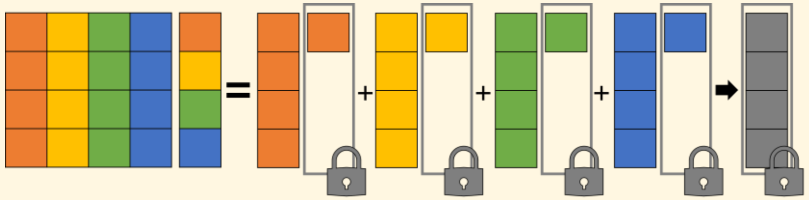

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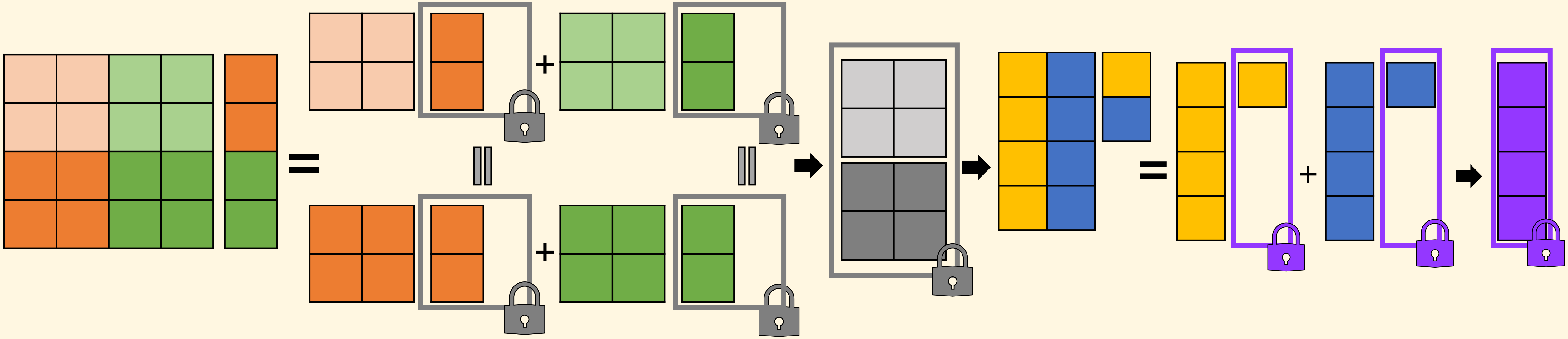

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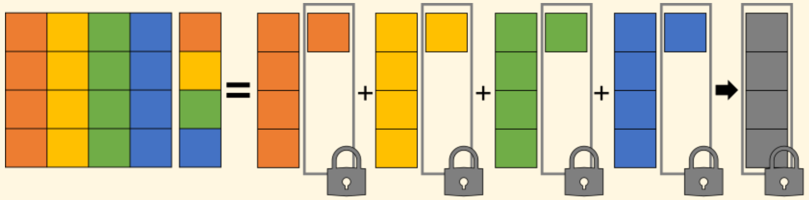

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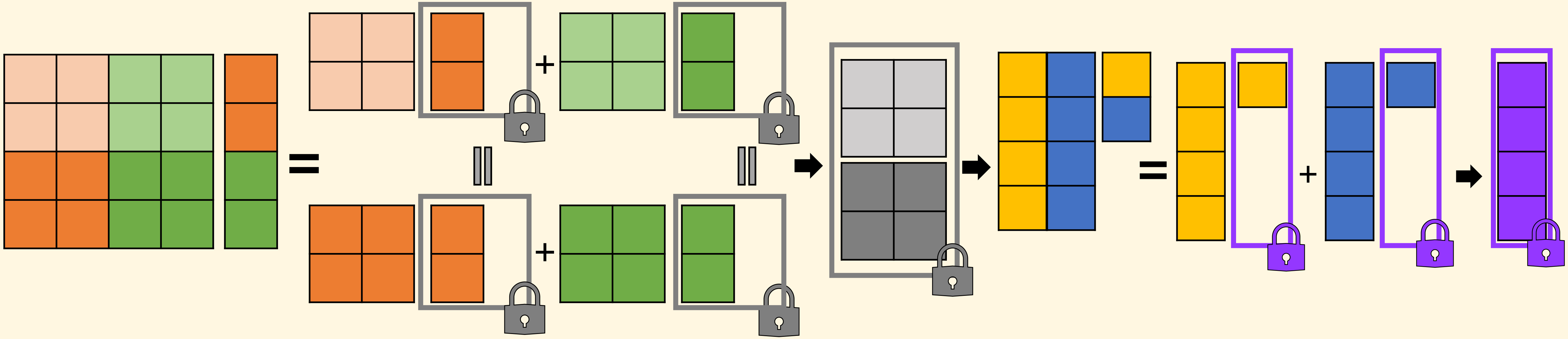

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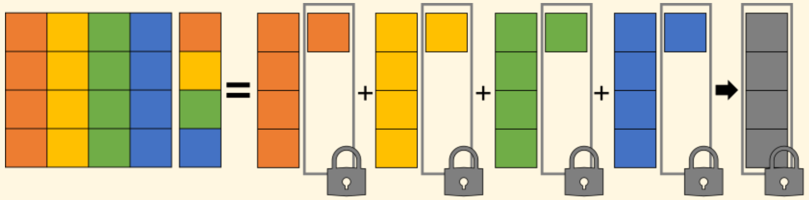

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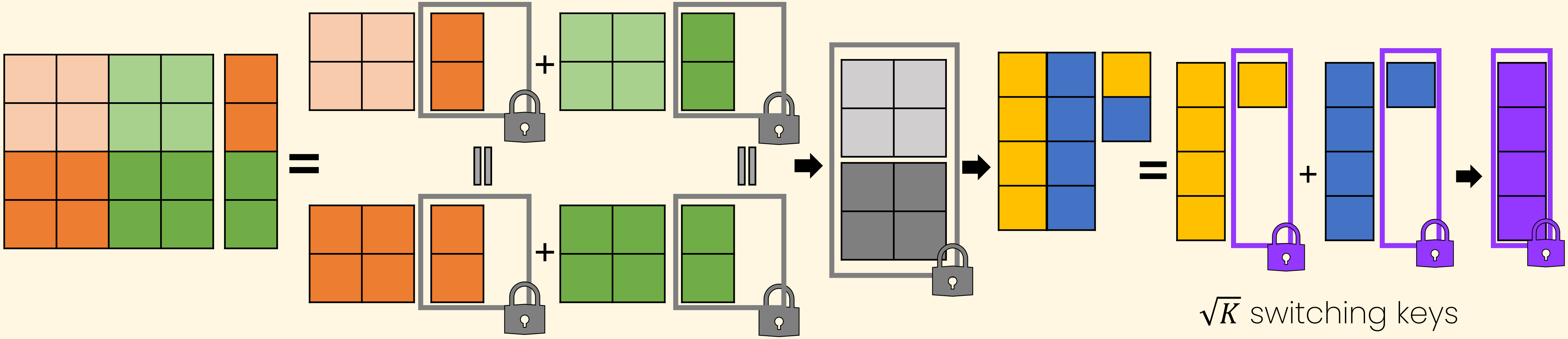

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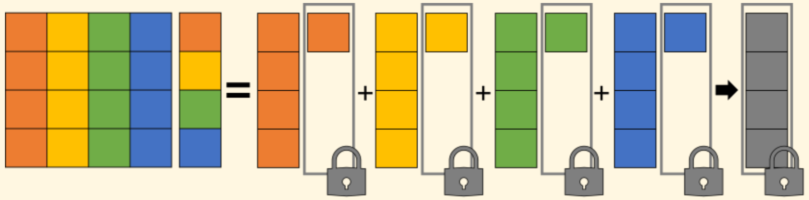

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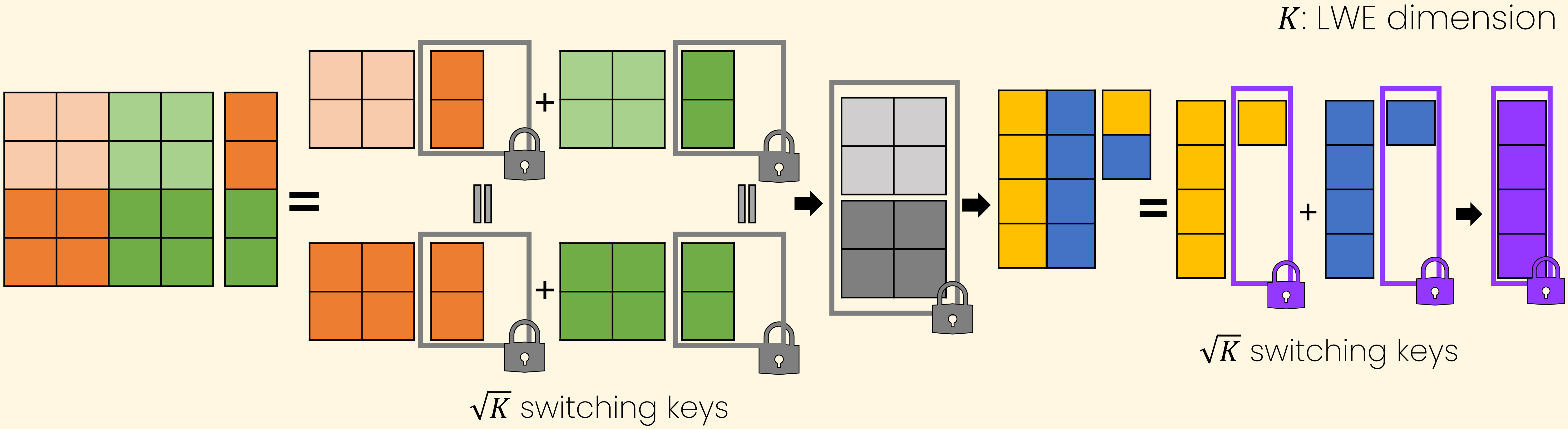

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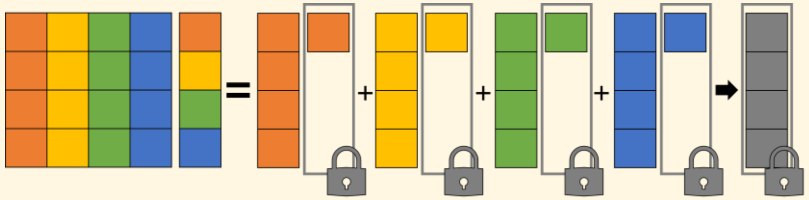

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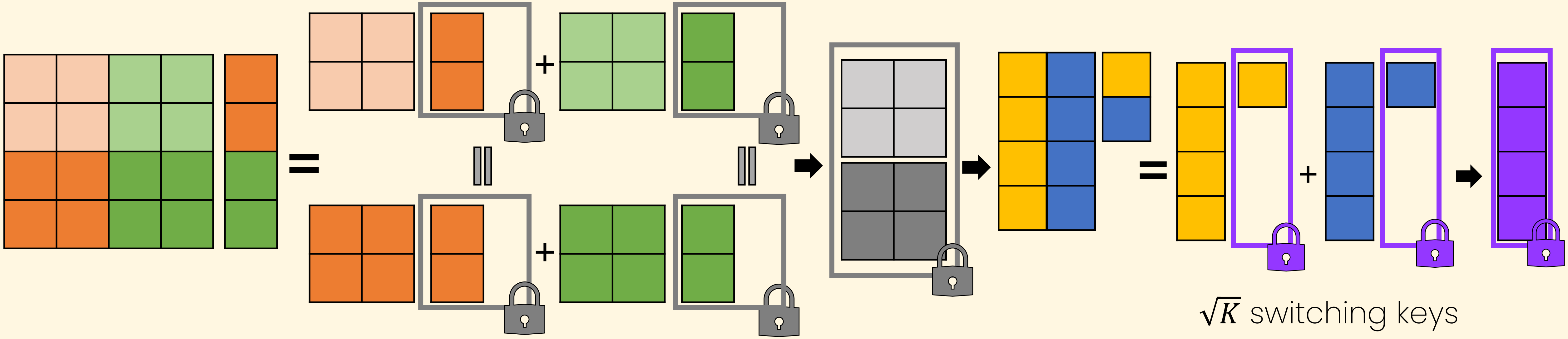

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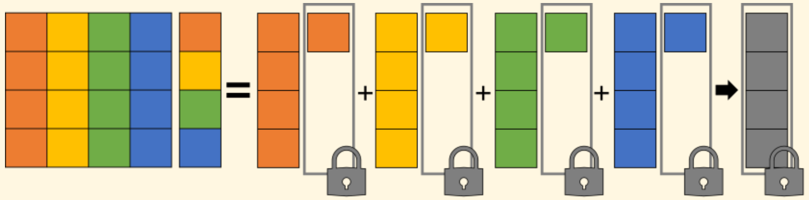

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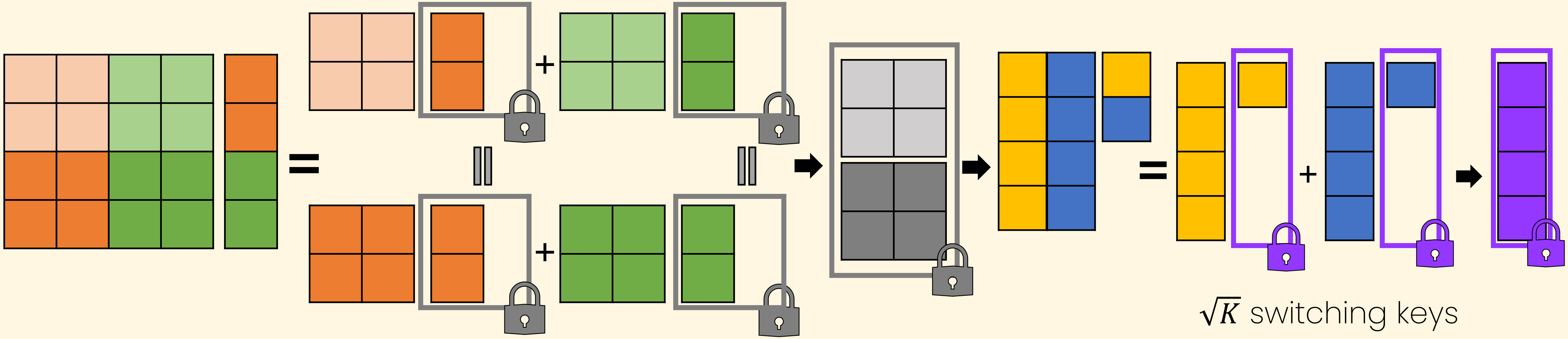

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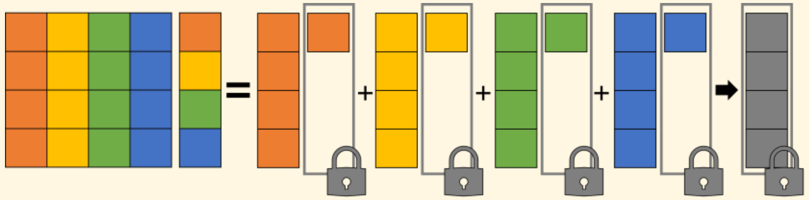

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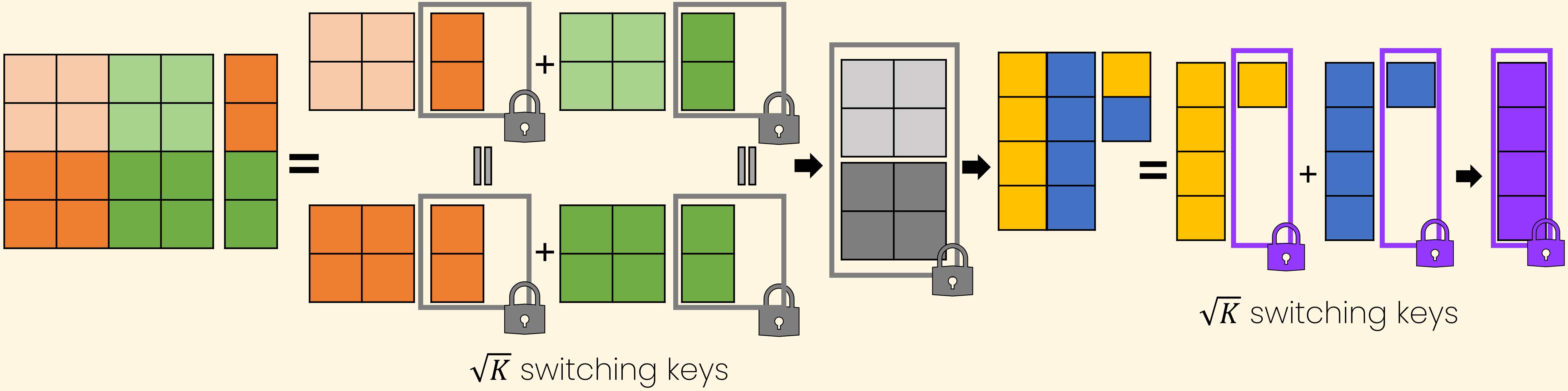

- How to encode the blocks? *Module LWE (MLWE)*
- How to squeeze the blocks? *MLWE key switching / MLWE ring switching*

Experimental Results

Ring Packing

<table>
<tr><th colspan="2" rowspan="2">FHE RP Method</th><th rowspan="2">RLWE modulus</th><th rowspan="2">RLWE degree</th><th rowspan="2"># of input LWE</th><th rowspan="2">Amortized Time (ms/slot)</th><th colspan="2">Key size (MB)</th></tr>
<tr><th>Base RP</th><th>Half-BTS</th></tr>
<tr><td>Pegasus [LHH+21]</td><td>Diagonal</td><td>2^{270}</td><td>2^{16}</td><td>2^{12}</td><td>12.62</td><td colspan="2">3540</td></tr>
<tr><td>[CDKS21] with Optimizations</td><td>Row</td><td>2^{562}</td><td>2^{16}</td><td>2^{16}</td><td>0.83</td><td>1</td><td>667</td></tr>
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All experiments are measured on AMD® Ryzen 7 3700x 8-core processor with a single-threaded CPU.
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Ring Packing

FHE RP Method		RLWE modulus	RLWE degree	# of input LWE	Amortized Time (ms/slot)	Key size (MB)	
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Transciphering

Scheme	RLWE degree	Granularity	Latency (s)	Expansion Ratio	Mean Precision
HERA [CHK+21]	2^{16}	16	141.6	1.24	19.1
Rubato [HKL+22]		64	106.4	1.26	18.9
		16	71.1	1.31	18.8
HERMES		64	25.8	1.58	23.0
		16	25.7	1.58	23.0
		1	30.9	1.58	23.0

All experiments are measured on AMD® Ryzen 7 3700x 8-core processor with a single-threaded CPU.
HERA and Rubato figures are borrowed from [CHK+21] and [HKL+22]; measured on AMD Ryzen 7 2700X @ 3.70 GHz single-threaded CPU.

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Thank you!

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